



How do I write that sum?

Handout Tuesday April 24, 2012

This is *not* the final handout for our class this semester. It is distributed in response to questions arising from the last homework assignment asking not about algebraic difficulties, but rather about how to write sums that include only the even-numbered terms or the odd-numbered terms from a finite sequence

$$a_0, a_1, a_2, \dots, a_n.$$

The sum of all terms in the sequence above can be written in many different ways, including:

$$a_0 + a_1 + a_2 + \cdots + a_n = \sum_{k=0}^n a_k = \sum_{0 \leq k \leq n} a_k.$$

The choice of which notation to use depends on personal preference, and is guided by concerns about what looks most natural or simpler. So I will not try to give you one way to write these sums, only a list of popular choices.

Sum of Terms with Even Index

We can always write $n = 2m$ or $n = 2m + 1$ (in the first case n is even; in the second case n is odd). The sum of the terms with *even* index k from 0 to n , can be written in several ways, including

$$a_0 + a_2 + a_4 + \cdots + a_{2m} = \sum_{\substack{0 \leq k \leq n \\ k \text{ even}}} a_k = \sum_{r=0}^m a_{2r}.$$

In the last sum, I have used r as a new dummy index of summation where $k = 2r$. These sums only make sense if m has been defined. The definition of m as given above is perfectly adequate, but alternative ways of expressing the same relationship between m and n are possible. For example, I could write

$$m = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is even;} \\ \frac{n-1}{2}, & \text{if } n \text{ is odd.} \end{cases}$$

We can also write simply $m = \lfloor \frac{n}{2} \rfloor$ where $\lfloor x \rfloor$ denotes the largest integer $\leq x$, i.e. we obtain $\lfloor x \rfloor$ by ‘rounding down’ x . Using this notation, we can write the sum of the terms with even index as

$$a_0 + a_2 + a_4 + \cdots + a_{2m} = \sum_{\substack{0 \leq k \leq n \\ k \text{ even}}} a_k = \sum_{r=0}^{\lfloor n/2 \rfloor} a_{2r}.$$

In most cases I would tend to use the middle choice above: simply sum over *even* values of k in the range $0 \leq k \leq n$, since it requires the least explanation.

Sum of Terms with Odd Index

How do we write the sum of the terms in the finite sequence $a_0, a_1, a_2, \dots, a_n$ with *odd* index? You can devise several possible ways to write this, by analogy with the case of even-numbered terms. In the end, the simplest is probably

$$\sum_{\substack{0 \leq k \leq n \\ k \text{ odd}}} a_k.$$