
# Affine Planes: Application to Experimental Design

Finite affine and projective planes were first studied by statisticians interested in their applications to *experimental design*. The following hypothetical experiment is greatly simplified in order to convey the general concept.

We imagine that a medical research firm wants to test the benefits of a new drug in lowering blood pressure. In this experiment the drug is to be administered at three dosage levels:

- Level I: 10 mg
- Level II: 5 mg
- Level III: 0 mg (in this case the patient is administered a placebo; this will be the control group)

One possible design for an experiment would be to randomly select several willing participants to the study and to divide the participants into three groups (I, II and III) of equal size. Participants in each of the three groups will be administered the drug at the corresponding level (I, II or III). No participant is informed which of the three groups they belong to, at least until the experiment is concluded. Over the course of one week of testing, the reduction in blood pressure for each individual is measured. An average is found for each of the three groups, and the results compared using standard statistical methods. Conclusions of the experiment should indicate whether or not the new drug is effective in reducing blood pressure; and the larger the number of participants in the experiment, the more confidence we will have in the results. Unfortunately such experiments are expensive, and the cost increases directly with the number of participants. Therefore experiments should be designed carefully in order to obtain the most reliable information with the fewest required number of trials.

The preceding simple scenario suffers from the following difficulty. Suppose that, in the interest of conserving cost, a small number of participants is selected; and suppose that by chance (as may very well happen) individuals in one of the three groups are predominantly heavier than those in another group. This may offset the results considerably, leading to incorrect conclusions. One possible solution would be to select participants only within a pre-selected narrow range of weights; but then we would not be testing whether the drug is beneficial to the general public, but rather to individuals only of a certain weight. A better solution is to make sure the three groups have a balance of individuals from each of the following three ranges of weight:

- Weight Range (a): under 120 lb.
- Weight Range (b): 120-175 lb.
- Weight Range (c): over 175 lb.

OK… maybe the age of the individual is also an important factor. We may also want to balance the three groups so that they have equal numbers of individuals in each of the following three age ranges:

- Age Range 1: under 40 yrs.
- Age Range 2: 40-60 yrs.
- Age Range 3: over 60 yrs.

And maybe we also want to balance the three groups so that each group has equal numbers of individuals from each of the following three ranges of income:

- Income Range A: under $15,000 per year
- Income Range B: $15,000-$50,000 per year
- Income Range C: over $50,000 per year

Here the suspicion is that an individual's income level may largely affect their diet, and therefore their response to treatment.

One way to be sure that we have balanced each of the three groups with individuals in each of the above ranges, as well as reducing the effects of various *combinations* of these factors, is to test 81 individuals with the following combinations of attributes and treatment:

- I-(a)-1-A (drug level I given to 1st individual, in weight range (a), age range 1 and income level A)
- I-(a)-1-B (drug level I given to 2nd individual, in weight range (a), age range 1 and income level B)
- I-(a)-1-C (drug level I given to 3rd individual, in weight range (a), age range 1 and income level C)
- I-(a)-2-A (drug level I given to 4th individual, in weight range (a), age range 2 and income level A)

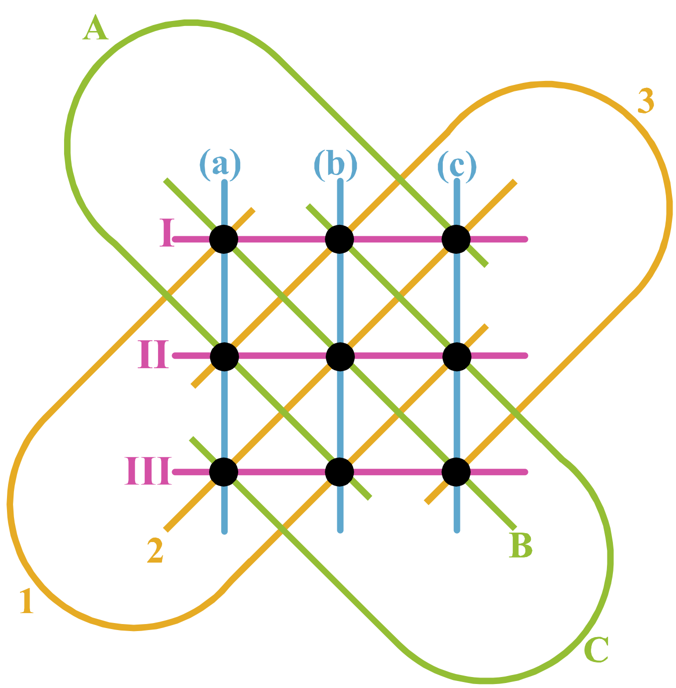
  ... *etc.* ...

- III-(c)-3-C (drug level III given to 81st individual, in weight range (c), age range 3 and income level C)

But this requires 81 individuals (or more if we try to test several individuals with each possible combination of attributes at each of the three levels of treatment) and it may in fact be very difficult to locate participants with all possible combinations of attributes.

A cheaper way which may well provide satisfactory balance is to test only nine individuals with the following combinations of attributes and treatments:

- I-(a)-1-B (drug level I given to 1st individual, in weight range (a), age range 1 and income level B)
- I-(b)-3-C (drug level I given to 2nd individual, in weight range (b), age range 3 and income level C)
- I-(c)-2-A (drug level I given to 3rd individual, in weight range (c), age range 2 and income level A)
- II-(a)-3-A (drug level II given to 4th individual, in weight range (a), age range 3 and income level A)
- II-(b)-2-B (drug level II given to 5th individual, in weight range (b), age range 2 and income level B)
- II-(c)-1-C (drug level II given to 6th individual, in weight range (c), age range 1 and income level C)
- III-(a)-2-C (drug level III given to 7th individual, in weight range (a), age range 2 and income level C)
- III-(b)-1-A (drug level III given to 8th individual, in weight range (b), age range 1 and income level A)
- III-(c)-3-B (drug level III given to 9th individual, in weight range (c), age range 3 and income level B)




Here we see the affine plane of order 3, in which the nine points are the nine individuals, and the twelve lines represent levels of treatment or ranges of a particular attribute. Parallel lines represent different ranges of the *same* attribute (or different levels of the drug treatment). If we test nine individuals with the indicated combinations of attributes over a one-week period, then a careful ANOVA (Analysis Of Variance) table will yield results as to how effective the drug was in lowering blood pressure, and indicate how the benefits vary for individuals with different attributes. Finite geometries of various sizes are tabulated and available for use in designing experiments so as to yield the greatest amount of information at the least possible cost.