Math 5605

Algebraic Topology

Book 1

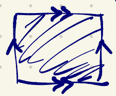
If X, Y are top. spaces, $f: X \to Y$ is <u>continuous</u> if $f^{-1}(U) \subseteq X$ is open whenever $U \subseteq Y$ is open.

$f: X \to Y$ is a <u>homeomorphism</u> if f is bijective and f, f^{-1} are continous.

$X \cong Y$ are <u>homeomorphic</u> if there exists a homeomorphism $X \xrightarrow{\cong} Y$.

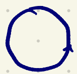
$\mathbb{R}^2 \not\cong S^2$ since S^2 is compact; $\mathbb{R}^2$ is not.

2-sphere $S^2 \cong \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$ = 

$S^2 \not\cong T^2 = S^1 \times S^1$ =  = 

(S^1 '' S)

S^2, T^2 are compact surfaces. They are locally homeomorphic but not globally homeomorphic.

$T^1 = S^1 =$  = circle $\cong \{z \in \mathbb{C} : |z| = 1\}$

$S^2 \not\cong T^2$ because S^2 is simply connected whereas T^2 is not.

In S^2, every closed path can be "continuously shrunk" to a point (homotopic to a point, i.e. null-homotopic)

$\not\cong$

although both surfaces are compact, connected, not simply connected

These two surfaces have different fundamental group: $\pi_1(X)$ is nonabelian, $\pi_1(T) \cong \mathbb{Z}^2$ is a (nontrivial) abelian group.

If $X \cong Y$ (homeomorphic) then $\pi_1(X) \cong \pi_1(Y)$.

For much of alg. top., the algebraic invariants that we define are actually invariant under the weaker equivalence relation of <u>homotopy equivalence</u>.

Eg. For every $n \geq 0$, $\mathbb{R}^n$ is homotopy equivalent to $\mathbb{R}^0 = \{\bullet\}$.