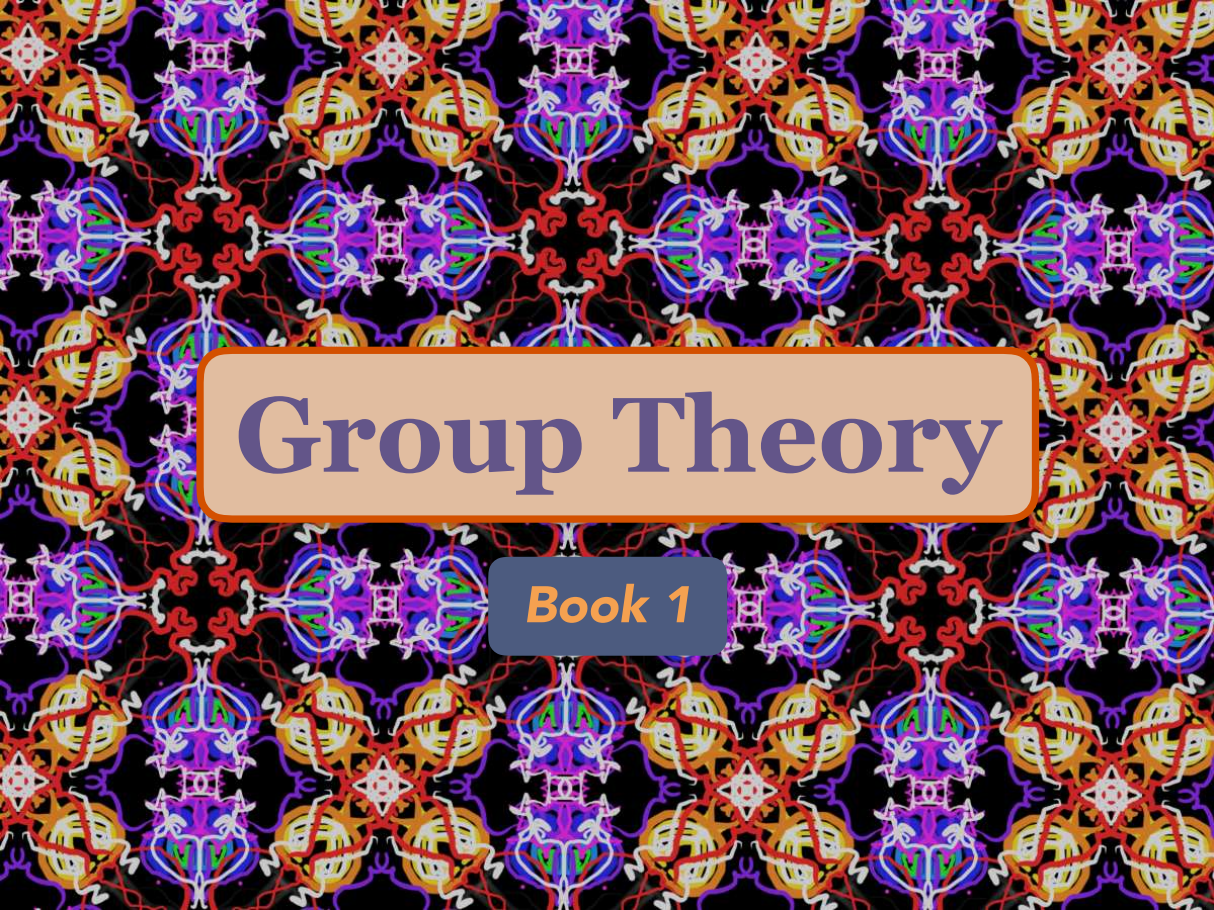
Group Theory
Book 1

Finite groups (up to isomorphism)

n	1	2	3	4	5	6	7	8	9	10	11	...	59	60	61	62	63	64	65	...
no. of groups of order n	1	1	1	2	1	2	1	5	2	2	1		1	13	1	2	1	267	1	

$C_2 \times C_2 \times C_3 \times C_5$
$C_4 \times C_3 \times C_5 \cong C_{60}$

$n=4$: Klein $C_2 \times C_2$, C_4
$n=6$: $C_3 \times C_2 \cong C_6$, $S_3 \cong D_3$
$n=8$: elementary abelian $C_2 \times C_2 \times C_2$, $C_2 \times C_4$, C_8, Q Quaternion, D_4 Dihedral
$n=9$: $C_3 \times C_3$, C_9 elementary abelian

S_n = symmetric group of degree n, $|S_n| = n!$
A_n = alternating group of degree n, order, $|A_n| = \frac{1}{2}n!$ $(n \geq 2)$

A_n is simple for $n \geq 5$
$|S_4| = 24$ solvable

$G_0 = 1 \overset{2}{\triangleleft} G_2 \overset{2}{\triangleleft} G_3 \overset{3}{\triangleleft} G_4 = A_4 \overset{2}{\triangleleft} G = S_4$

$G_2 = \langle (12)(34) \rangle$ order 2
$G_3 = \langle (12)(34), (13)(24) \rangle$ order 4

Composition series with composition factors of prime orders: $|G_1/G_0| = 2$, $|G_3/G_2| = 2$,
$|G_4/G_3| = 3$, $|G/G_4| = 2$

G is solvable if all its composition factors are cyclic of prime order.

Jordan-Hölder Theorem: Every finite group has a composition series with its factors being simple groups.

G $(G \neq 1)$ is simple if its only composition series is $1 \triangleleft G$ (the only normal subgroups are 1 and G).

eg. cyclic groups of prime order are simple.
A_n is simple for $n \geq 5$.