HW2

(Due Monday, November 11, 2019)

Instructions: See the course syllabus for general expectations regarding homework. Note especially that you may discuss the problems with other students, but you should write up solutions yourself. *If your work makes use of explicit multiplication of permutations, indicate clearly whether you are using left-to-right or right-to-left composition of permutations.*

1. Consider the group given by the presentation $G = \langle a, b \,:\, a^5 = b^3 = (ab)^2 = 1 \rangle$.
   (a) Use Coxeter-Todd coset enumeration to determine $|G|$. (Show your work as we have done in the examples worked in class. The example of $B(2,3)$ is available on the course website.)
   (b) Explain how you obtain upper and lower bounds for $|G|$ which agree, thereby determining the exact value of $|G|$.
   (c) Using your work in (a), give a pair of permutations, satisfying the given relations, which generate G.

   (After completing this problem by hand, it is well worth your time to verify your computations using appropriate software such as GAP, as we demonstrate in class. Other suitable choices of software include Magma, Maple and Mathematica.)

2. Consider the permutation group $G = \langle \rho, \sigma \rangle \leqslant S_{14}$ where $\rho = (1,3,5,7,9,11,13)(2,4,6,8,10,12,14)$ and $\sigma = (1,2)(3,6)(4,5)(7,8)(9,12)(10,13)(11,14)$. *For this problem you may use computer and/or hand computation.*
   (a) Determine $|G|$.
   (b) Find a presentation of G with two generators and as few relations as possible, such that ρ and σ satisfy the relations you have chosen.

3. Find positive integers m and n such that the Burnside group $B(m,n)$ has *both* of the groups above (in #1 and #2) as homomorphic images. Explain. (You must find m and n such that $B(m,n)$ has normal subgroups K_1 and K_2, such that $B(m,n)/K_1$ and $B(m,n)/K_2$ give the groups in #1 and #2 respectively. Explicit determination of K_1 and K_2, however, is not necessary. Likewise, the explicit determination of $B(m,n)$ is not required; you shouldn't even worry whether this Burnside group is finite or infinite.)