A 2-Hats Problem

Two participants are gathered for a game in which I place on each participant's head a cap which is either red or blue. I assign hat colours according to my own preferences (possibly randomly).

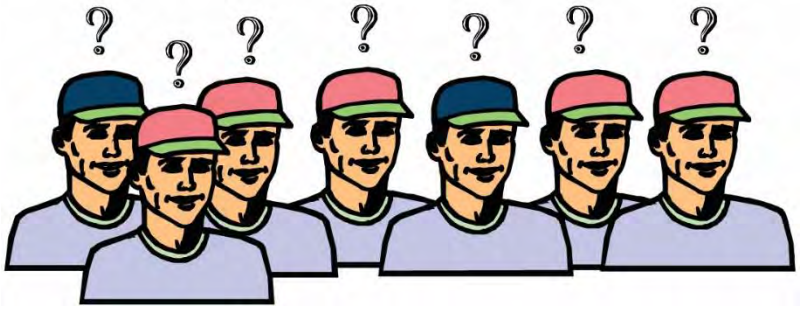
Each participant can see the cap on the other participant's heads but cannot see their own cap. Each participant must guess the colour of their own cap. If at least one participant guesses correctly, I suitably reward them both; otherwise (if both guess wrong) then I punish both of them by death.

Before the game starts, the participants are allowed to confer to come up with a strategy for guessing. But once the game starts and I place the caps on their heads, no further communication between them is permitted.

What strategy maximizes the probability that the participants succeed?

A 7-Hats Problem

Seven participants are gathered for a game in which I place on every participant's head a cap which is either red or blue. I assign hat colours randomly (each hat red or blue, each with probability 50%, and all hats chosen independently).



Each participant can see the caps on the other six participants' heads but cannot see their own cap. I ask each participant to guess the colour of their own cap, and they can answer 'red', 'blue' or 'pass'. I judge as follows:

- FAILURE occurs if every participant passes, or if any participant guesses the colour of their hat 'red' or 'blue' incorrectly. In this case, I punish all participants by death.
- SUCCESS occurs if at least one participant guesses a colour, and every participant who answers 'red' or 'blue' correctly guesses their hat colour. In this case, I suitably reward all participants.

What strategy maximizes the probability that the participants succeed?