Test 1—Thursday, October 18, 2017

Instructions: Answer all questions in the space provided. You are permitted to use one "cheat sheet" (one side of an 8½×11 in. sheet with your own handwriting only) and a calculator. Show complete work and use complete sentences when required. Total value of questions: 100 points.

Section A: True/False *(3 points each)*

1. Due to the work of Hilbert and other leading mathematicians of the 20th century, the mathematical problems of Euclidean geometry are now solved, or at least algorithmically solvable using computers. ___________________ *(True/False)*

2. Given any projective plane, if one removes a quadrilateral, what remains is an affine plane. ___________________ *(True/False)*

3. The axioms of plane geometry as taught in high school today are found in the writings of Euclid. ___________________ *(True/False)*

4. Every projective plane contains a set of four lines, no three of which are concurrent. ___________________ *(True/False)*

5. In affine n-space F^n over an arbitrary field F, where $n \geq 2$, any three points lie on at least one plane. ___________________ *(True/False)*

6. The one- and two-dimensional subspaces of $\mathbb{R}^3$ form the points and lines of a projective plane. ___________________ *(True/False)*

7. In affine plane geometry, a line segment is defined as the shortest path between two points. ___________________ *(True/False)*

8. In affine plane geometry, every point is incident with at least three lines. ___________________ *(True/False)*

9. In the axiomatic approach to plane geometry, the term '*line*' is defined as an algebraic curve of degree one. ___________________ *(True/False)*

10. In the game *SpotIt*®, cards and symbols represent points and conics of an affine plane. ___________________ *(True/False)*

Section B: Classical Plane Geometry

Instructions: In #11, fill in each of the blanks below using the *best* word selected from the following list. You may use a word more than once, or not at all.

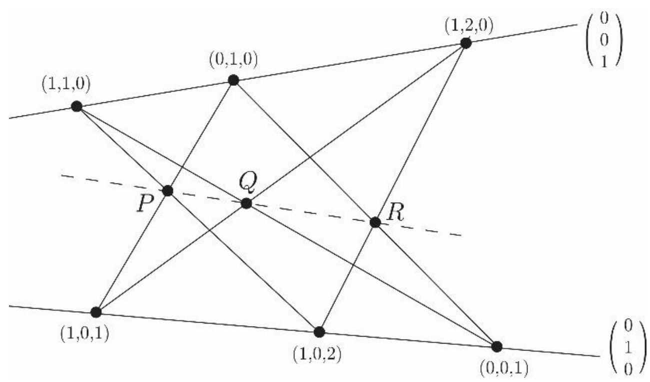
plane	triangle	axioms	conic	point	distance	classical
quadrilateral	concurrent	algebra	polarity	line	angle	affine
Pappus	collinear	parallel	infinity	determinant	order	projective

11. *(32 points)* The classical theorem of ______________ holds in the Euclidean plane only if one is careful to state the exceptional cases that arise due to the possibility of ____________ lines. If one adds a point at _______________ to every _____________ class of lines in the Euclidean plane, then joins all the new points with a single new ______________, one obtains a classical ______________ plane. In this new plane, the classical theorem of _____________ holds without exception because no two lines are ______________.

The Euclidean plane is an example of a classical ______________ plane; but because it is coordinatized by the real numbers, it admits several features not found in a more general ______________ plane. Examples of these properties include _____________ between two points; the _____________ of points on a line; and the ______________ between two intersecting lines.

In the Euclidean plane, a ____________ has the property that no three of its points are ________________. This property can be proved, not directly from the axioms, but rather using ______________.

12. *(16 points)* The figure on the right depicts a configuration of points and lines in the real projective plane. Determine the coordinates of the points P, Q, R and verify their collinearity.



13. *(12 points)* Consider the following statement, valid theorem in the theory of projective planes:

Theorem. *Suppose a plane has the following property: Given any four points A, B, C, D, no three of which are collinear, then the points $P = AB \cap CD$, $Q = AC \cap BD$ and $R = AD \cap BC$ are collinear. Then the plane is classical.*

State the dual of this theorem, which is also a valid theorem. (Do not supply any proof.)

Section C: Fractals

14. *(10 points)* The Sierpinski Triangle S is the limit of the following infinite sequence of plane regions, each having area $\frac{3}{4}$ times the area of the previous region in the sequence. Determine the Hausdorff dimension of S.