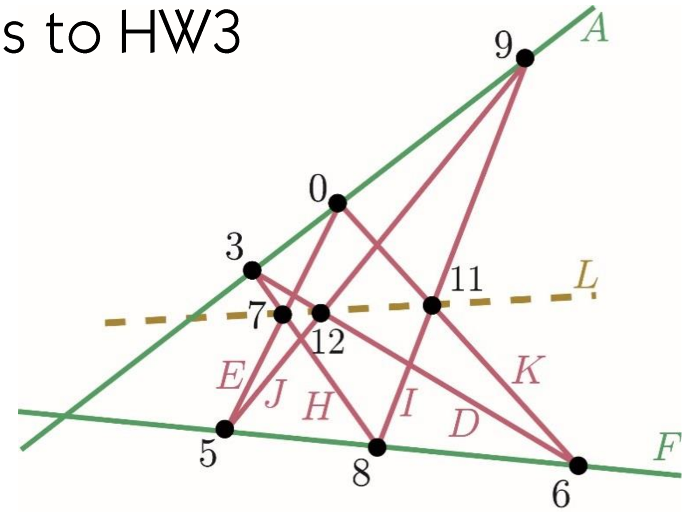
University of Wyoming — Math 4600—Fall 2018 — Department of Mathematics

Foundations of Geometry

Solutions to HW3



1. The dashed line L does pass through the three points 7,11,12 as required by the dual of Pappus' Theorem.

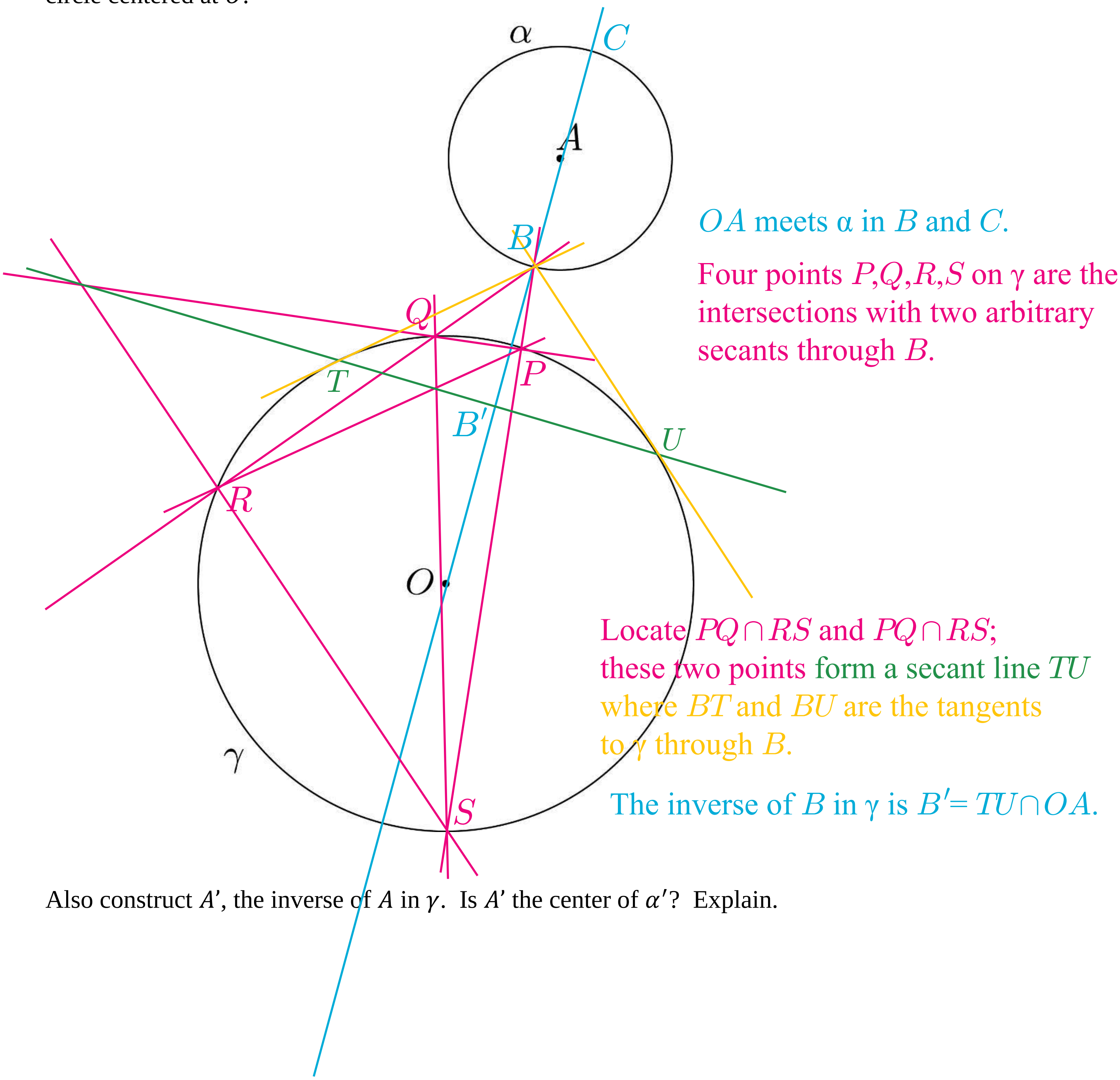
2. For the conic $S = \{0, 1, 2, 6\}$ in the projective plane of order 3 shown:
   a. The passant lines are H, I, J.
   b. The tangent lines are C, D, E, L.
   c. The secant lines are A, B, F, G, K, M.
   d. The interior points are 8, 9, 10.
   e. The absolute points are 0, 1, 2, 6.
   f. The exterior points are 3, 4, 5, 7,11, 12.
   g. There are 3 passant lines, each of which passes through 2 interior points, 0 absolute points and 2 exterior points.
      There are 4 tangent lines, each of which passes through 0 interior points, 1 absolute point and 3 exterior points.
      There are 6 secant lines, each of which passes through 1 interior point, 2 absolute points and 1 exterior point.
   h. There are 3 interior points, each of which lies on 2 passant lines, 0 tangent lines and 2 secant lines.
      There are 4 absolute points, each of which lies on 0 passant lines, 1 tangent line and 3 secant lines.
      There are 6 exterior points, each of which lies on 1 passant line, 2 tangent lines and 1 secant lines.
   i. Corresponding blanks in (g) and (h) are filled in with the same numbers. This bears out the principle of duality which interchanges

$$\text{interior points} \leftrightarrow \text{passant lines}$$
$$\text{absolute points} \leftrightarrow \text{tangent lines}$$
$$\text{exterior points} \leftrightarrow \text{secant lines}$$

3.

+	O	(0,0)	(1,0)	(6,0)	(5,1)	(5,6)	(4,2)	(4,5)
O	O	(0,0)	(1,0)	(6,0)	(5,1)	(5,6)	(4,2)	(4,5)
(0,0)	(0,0)	O	(6,0)	(1,0)	(4,2)	(4,5)	(5,1)	(5,6)
(1,0)	(1,0)	(6,0)	O	(0,0)	(5,6)	(5,1)	(4,5)	(4,2)
(6,0)	(6,0)	(1,0)	(0,0)	O	(4,5)	(4,2)	(5,6)	(5,1)
(5,1)	(5,1)	(4,2)	(5,6)	(4,5)	(1,0)	O	(6,0)	(0,0)
(5,6)	(5,6)	(4,5)	(5,1)	(4,2)	O	(1,0)	(0,0)	(6,0)
(4,2)	(4,2)	(5,1)	(4,5)	(5,6)	(6,0)	(0,0)	(1,0)	O
(4,5)	(4,5)	(5,6)	(4,2)	(5,1)	(0,0)	(6,0)	O	(1,0)

4. The circle α is centered at A as shown. Construct the image α' of α under inversion in γ, a circle centered at O.



Also construct A', the inverse of A in γ. Is A' the center of α'? Explain.

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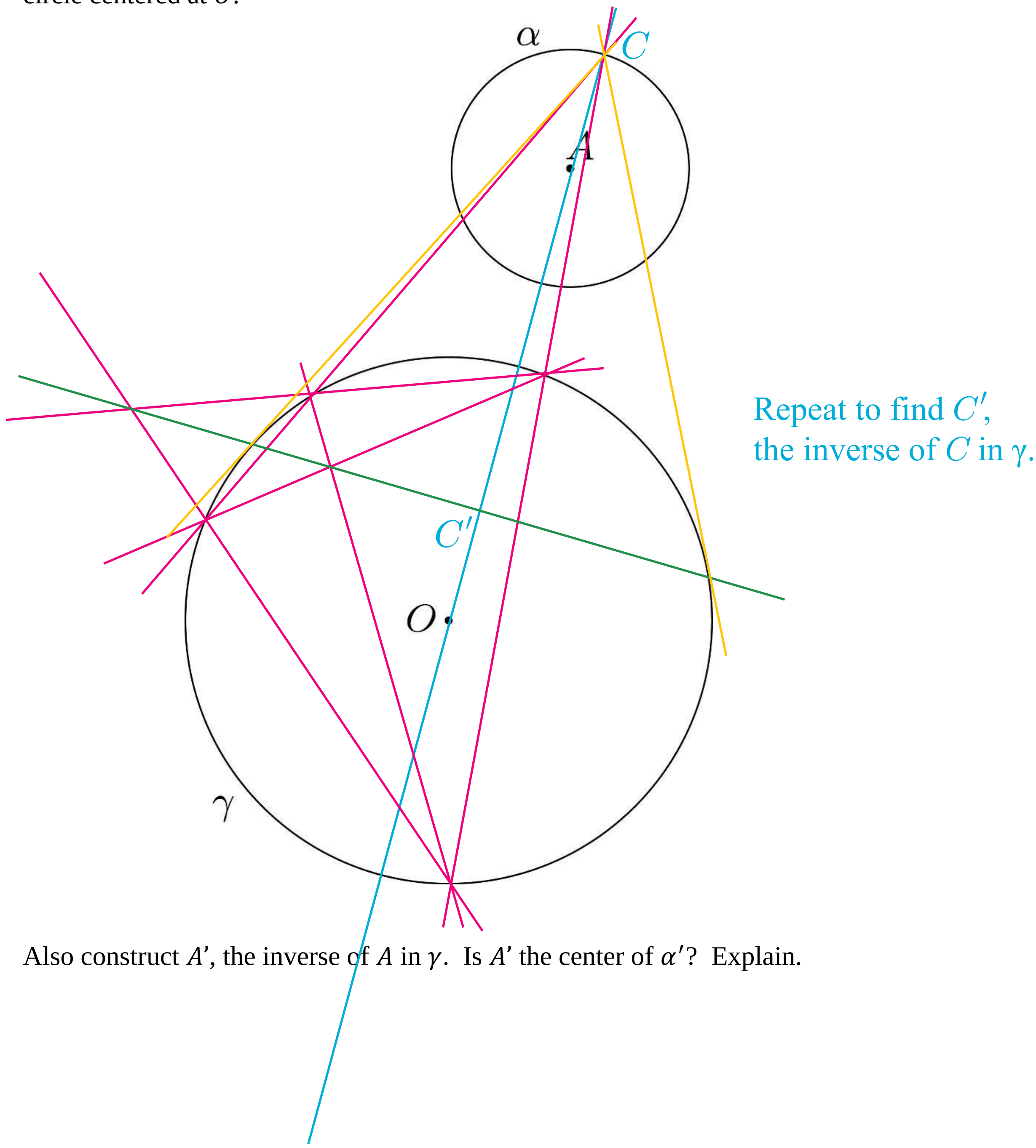

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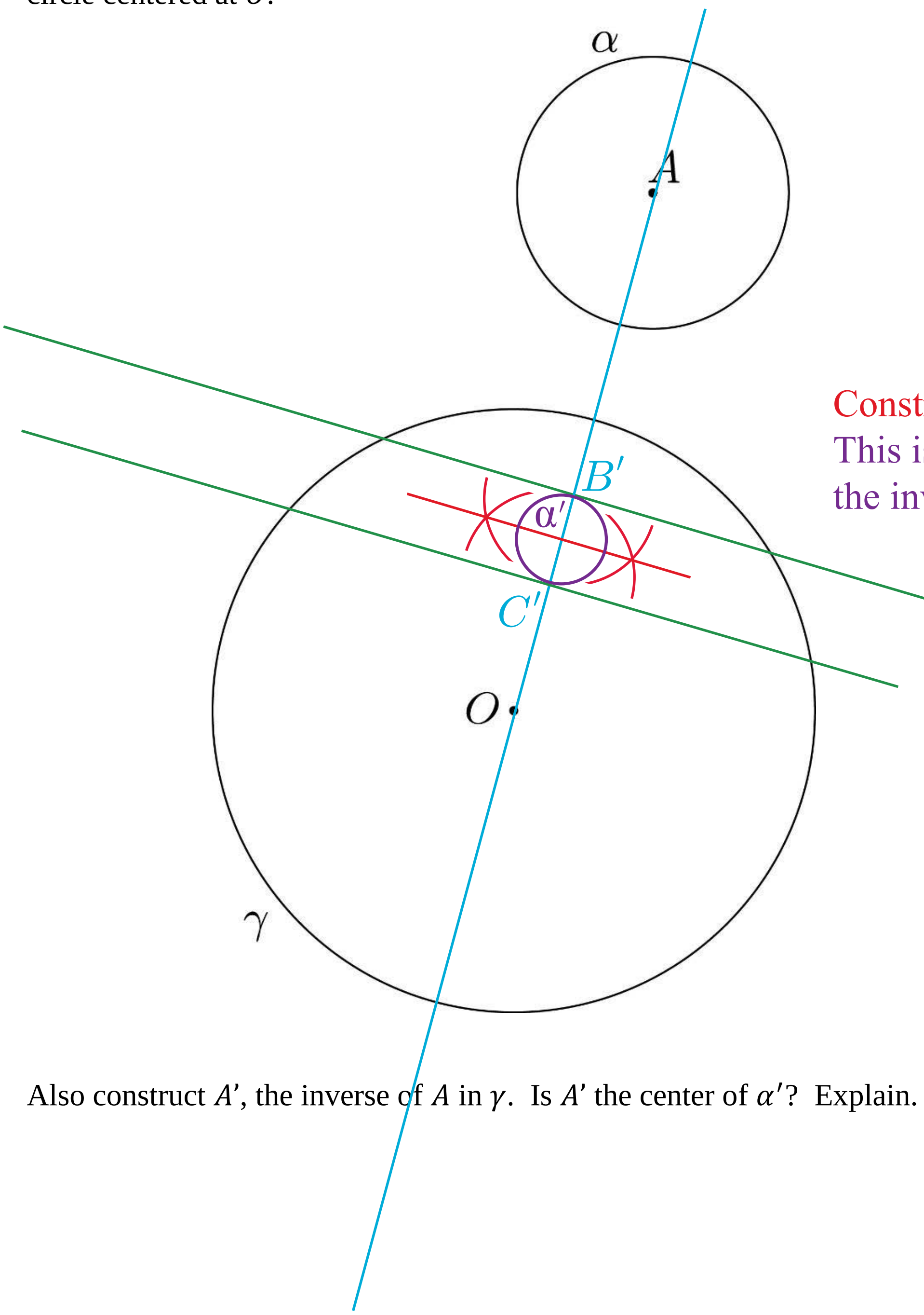
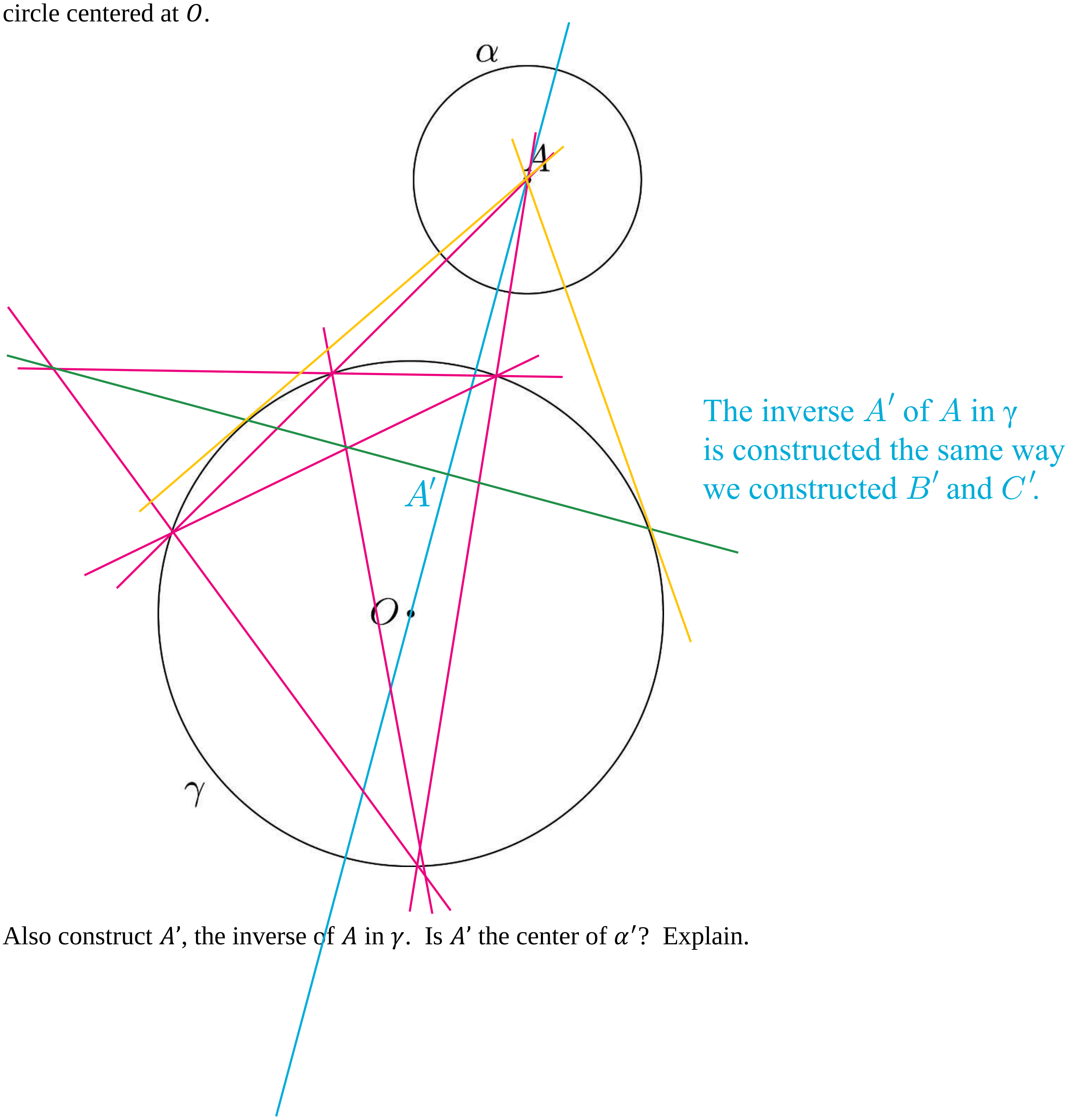


Construct the midpoint of $B'C'$.
This is the center of α′,
the inverse of α.

Also construct A', the inverse of A in γ. Is A' the center of α'? Explain.

4. The circle α is centered at A as shown. Construct the image α' of α under inversion in γ, a circle centered at O.



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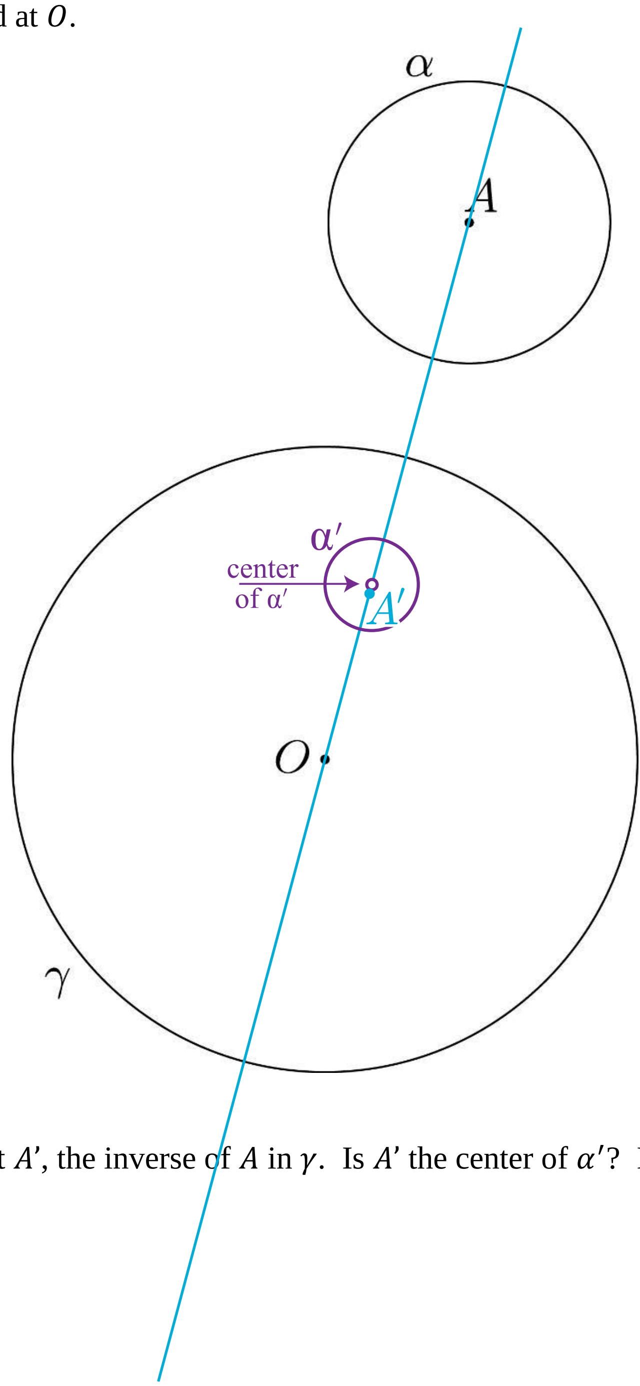
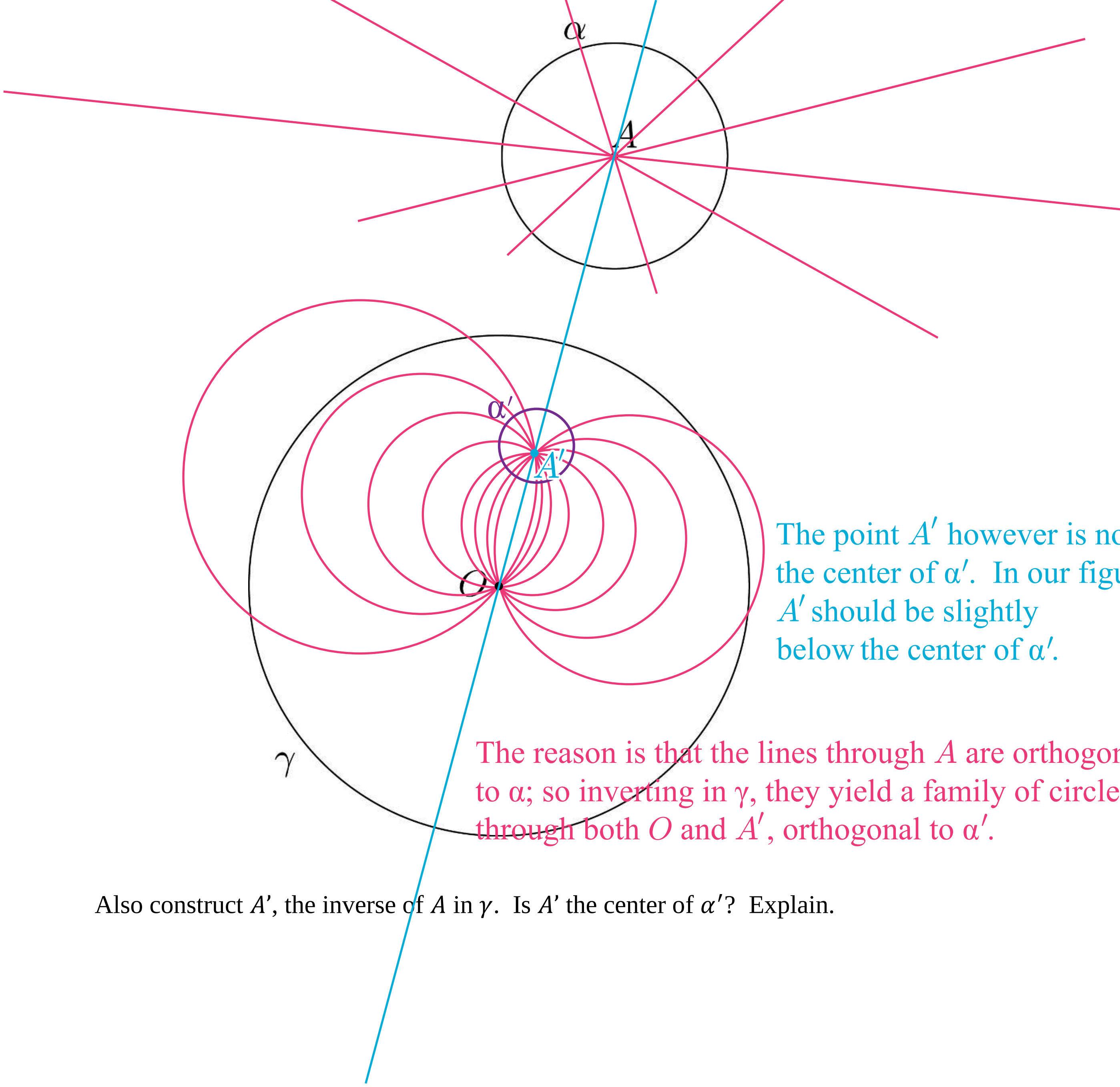


The point A' however is not the center of α'. In our figure, A' should be slightly below the center of α'.

Also construct A', the inverse of A in γ. Is A' the center of α'? Explain.

4. The circle α is centered at A as shown. Construct the image α' of α under inversion in γ, a circle centered at O.

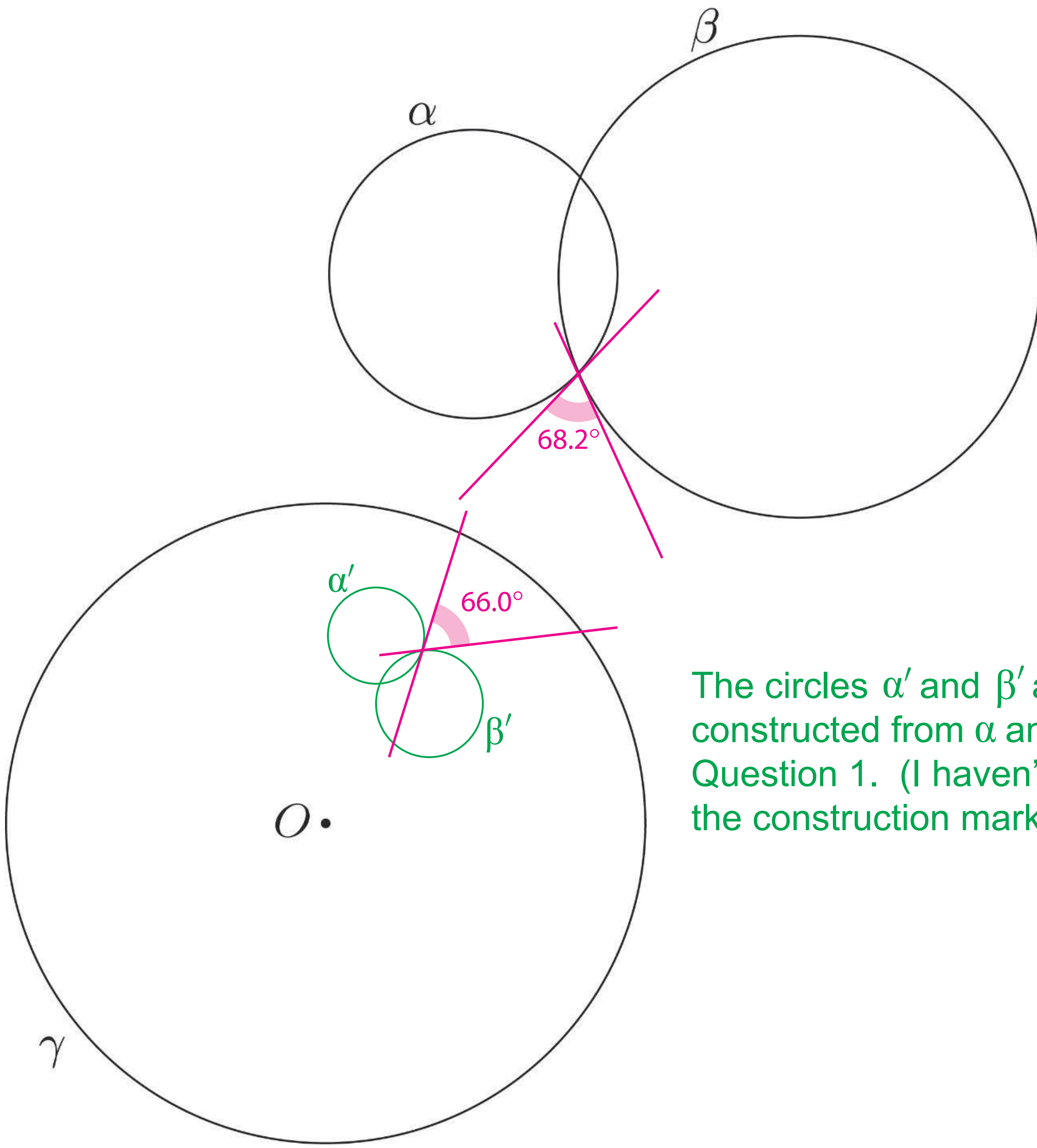


The point A' however is not the center of α'. In our figure, A' should be slightly below the center of α'.

The reason is that the lines through A are orthogonal to α; so inverting in γ, they yield a family of circles through both O and A', orthogonal to α'.

Also construct A', the inverse of A in γ. Is A' the center of α'? Explain.

5. Construct the inverses α', β' (respectively) of the circles α, β (as shown) in the circle γ (centered at O).



The circles α′ and β′ are constructed from α and β as in Question 1. (I haven't shown all the construction marks.)

Measure (as well as you can using a protractor) the angle between circles α and β. (This requires first drawing tangent lines to α and β at a point of intersection.) Do the same for α' and β'.

angle between α and β = about 68°

angle between α' and β' = about 66°

How do these two angles compare?

The angles are close as measured; any discrepancy is presumably due to errors in construction and measurement.