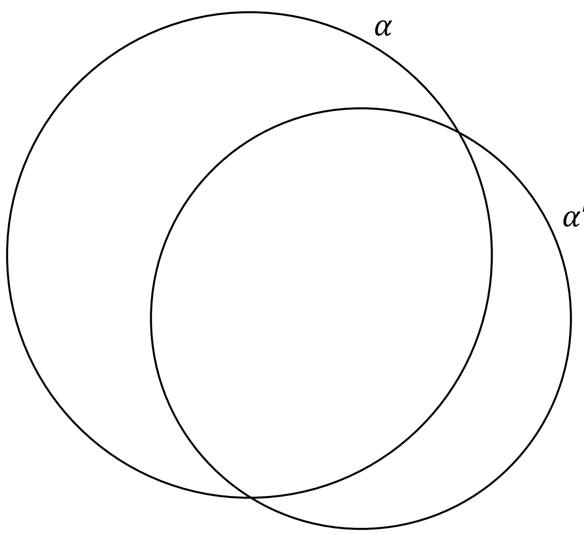
Final Exam (SAMPLE ONLY)

This sample exam is intended to resemble the Final Exam in approximate length, difficulty, and style, although clearly the content may differ. Although the actual exam copy will provide sufficient blank space for most answers, here I have omitted this blank space in order to save paper. The actual exam will cover all material covered this semester in class (including handouts and homework), with greater emphasis on the more recent material. The actual exam will be written in our usual classroom (BU 208) at 1:15–3:15pm on Thursday, December 13, 2018.

Instructions: Answer all questions in the space provided (or attach additional pages as needed). You are permitted to use pencils/pens, one "cheat sheet" (one side of an 8½″×11″ sheet with your own handwriting only), and a calculator. No sharing of calculators, or use of cellphones or audio listening devices, will be permitted. Show complete work and use complete sentences when required. Total value of questions: 100 points.

1. (*10 points*) Two intersecting circles α and α' are shown. Does there exist a circle γ inverting α to α'? Justify your answer.



2. (*10 points*) Consider the algebraic curve $y^2 = x^3 - x$ in the Euclidean plane.
   a. What is the degree of this curve?
   b. What is the maximum number of collinear points on the curve? Explain your answer.

3. (*10 points*) Find the equation of the unique conic in the real projective plane passing through the five points $(1,0,0)$, $(0,1,0)$, $(0,0,1)$, $(1,-1,1)$, $(1,2,4)$.

4. (*10 points*) State at least one difference between
   a. the Euclidean plane and the hyperbolic plane;

   b. the Euclidean plane and a plane in physical space (i.e. a plane in the universe we live in);

   c. the hyperbolic plane and a plane in physical space.

5. (*10 points*) Write down the three axioms for projective plane geometry. Then state and prove the dual of the third axiom.

6. (*10 points*) Explain clearly how every affine plane may be extended to a projective plane by the addition of 'points at infinity'. In what sense are projective planes more natural objects of study than affine planes?

7. (*10 points*) Briefly (using about a paragraph) but clearly, describe an example of an application of a finite geometry.

8. *(30 points)* Answer *True* or *False* to each of the following statements.
   a. For any surface S in Euclidean 3-space, any two geodesics meet in at most one point.
   ___________________ *(True/False)*

   b. The hyperbolic plane may be tiled with triangles having angles 30°-45°-90°.
   ___________________ *(True/False)*

   c. The dual of an inversive plane (obtained by interchanging the notions of 'point' and 'circle') is again an inversive plane. ___________________ *(True/False)*

   d. The axioms of Euclidean plane geometry are known to be inconsistent.
   ___________________ *(True/False)*

   e. Performing two successive inversions in the real inversive plane, has the same effect as a single inversion. (That is, composing two inversions yields an inversion.)
   ___________________ *(True/False)*

   f. Given three distinct collinear points A, B, C in the real projective plane, exactly one of the points A, B, C lies between the other two. ___________________ *(True/False)*

g. In the hyperbolic plane there exists a polygon with area greater than 100.

____________________ *(True/False)*

h. A theorem of Euclidean plane geometry states that on every simple closed curve, one can always find four points which form the vertices of a square.

____________________ *(True/False)*

i. Stereographic projection, from a sphere to a plane, distorts both distances and angles.

____________________ *(True/False)*

j. The real inversive plane is orientable. ____________________ *(True/False)*