Test 1 (SAMPLE ONLY)

This sample test is intended to resemble Test 1 in approximate length, difficulty, and style, although clearly the content may differ. The actual test will cover all material covered this semester in class (including lectures, handouts and homework). The actual test will be written during class time on Thursday, October 18.

Instructions: Answer all questions in the space provided. You are permitted to use one "cheat sheet" (one side of an 8½×11 in. sheet with your own handwriting only) and a calculator. Show complete work and use complete sentences when required. Total value of questions: 100 points.

Section A: True/False *(3 points each)*

1. The axioms for projective planes are complete. __________________ *(True/False)*

2. The axioms for projective planes are consistent. __________________ *(True/False)*

3. The Euclidean plane is an affine plane. __________________ *(True/False)*

4. Euclid's axioms of plane geometry are justified by the fact that they are universally valid truths, as is evident in the physical universe in which we live.
__________________ *(True/False)*

5. If one starts with a projective plane and removes any of its lines (together with all points on that line), what is left is necessarily an affine plane.
__________________ *(True/False)*

6. The Theorem of Pappus is deduced from the axioms of affine plane geometry using a direct proof in which every statement follows either from the axioms, or from a previous step in the proof.
__________________ *(True/False)*

7. As axioms for Euclidean plane geometry, one can take the very simple axioms of affine plane geometry, and then add certain additional axioms governing distance, angle, betweenness, etc.
__________________ *(True/False)*

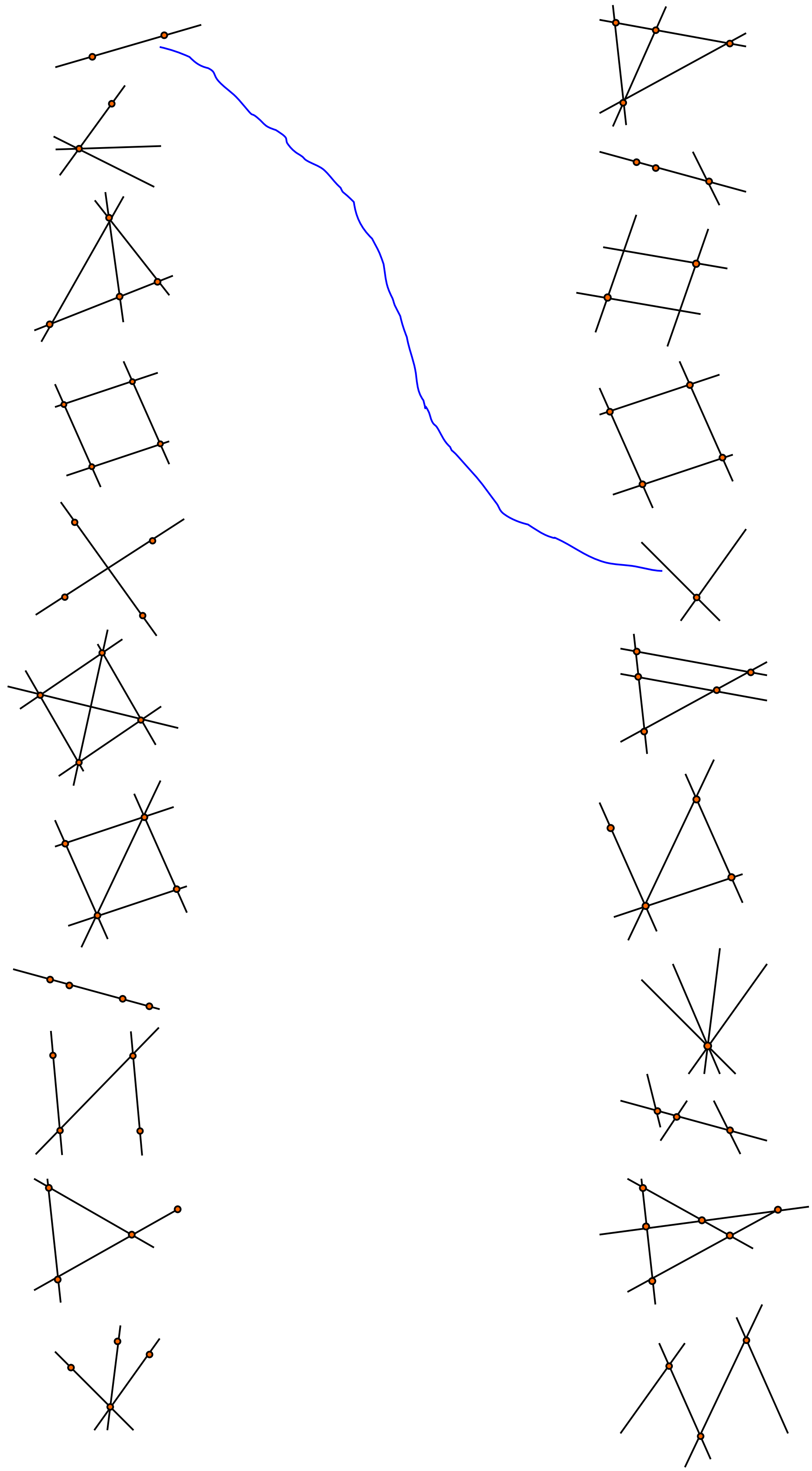
8. In the axiomatic approach to plane geometry, the term 'point' is defined as an object with position but without size.
__________________ *(True/False)*

9. Given any point P and line ℓ in an an affine plane, there is a unique line through P perpendicular to ℓ.
__________________ *(True/False)*

10. Unlike some areas of higher mathematics, Euclidean plane geometry harbors no open questions: existing methods allow us to resolve every statement as either true or false.
__________________ *(True/False)*

Section B

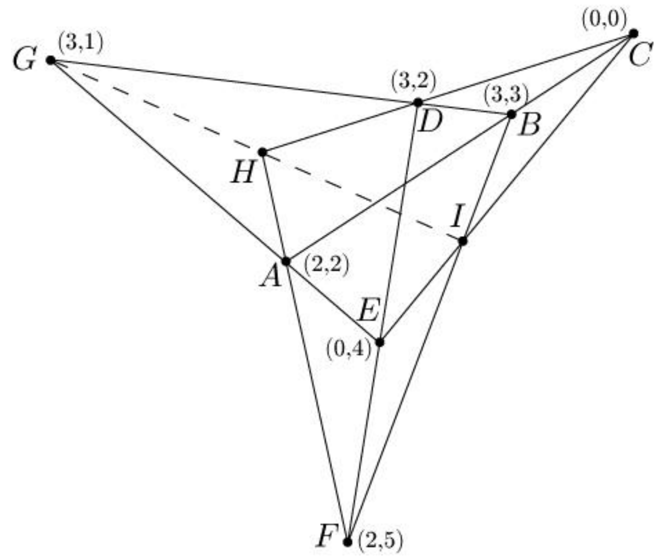
11. *(10 points)* Match each plane object on the left with its point-line dual on the right. The first one is done for you.



12. *(10 points)* Let $A \subset [0,1]$ be the set of real numbers between 0 and 1 having a decimal expansion with only even digits (for example $0.244862002 \ldots \in A$ but $0.8216103 \ldots \notin A$). Determine the Hausdorff dimension of A.

13. *(15 points)* Describe at least two advantages of projective plane geometry over affine plane geometry.

14. *(20 points)* The figure at the right shows a configuration of points and lines in a classical affine plane. There is a theorem of classical affine plane geometry asserting that if the triples of points ABC, DEF, AEG, AFH, BDG, BFI, CDH and CEI are collinear (as shown by the solid lines in the figure), then the triple GHI is also collinear (as shown by the dashed line in the figure).



    a. Name the theorem, stated in class, which makes this assertion.
    b. Assuming that this is the classical plane defined over the field of order 7 (the integers mod 7) and that A, B, C, D, E, F, G have coordinates as shown, find the missing coordinates of the points H and I. Then verify that G, H, I are indeed collinear.

15. *(15 points)* Show that there is an affine plane whose points are the ordered pairs of real numbers $(x, y) \in \mathbb{R}^2$ with $y > 0$ (usually referred to as the 'upper half-plane'). (Note that since the point set is already specified, what remains is for you to define the lines in such a way that the axioms of affine plane geometry are satisfied.)