Optional Retake of Test 1—Thursday, November 8, 2018

Instructions: Answer all questions in the space provided. You are permitted to use one "cheat sheet" (one side of an 8½×11 in. sheet with your own handwriting only) and a calculator. Show complete work and use complete sentences when required. Total value of questions: 100 points.

Section A: True/False *(3 points each)*

1. The number system $\mathbb{R} \cup \{\infty\}$, consisting of the real numbers together with a single extra quantity called 'infinity', is a field which can be used to coordinatize the Euclidean plane. ___________________ *(True/False)*

2. Given any projective plane, if one removes a line together with its points, what remains is necessarily an affine plane. ___________________ *(True/False)*

3. The Euclidean plane is an example of an affine plane. ___________________ *(True/False)*

4. Given any three stars in the physical universe, there is a unique Euclidean plane containing their centers. ___________________ *(True/False)*

5. In affine n-space F^n over an arbitrary field F, where $n \geq 2$, any three points lie on at least one plane. ___________________ *(True/False)*

6. If F is any field, then the one- and two-dimensional subspaces of the three-dimensional vector space F^3 form the points and lines of a projective plane. ___________________ *(True/False)*

7. In affine plane geometry, we define a point as 'that which has position but no size'. ___________________ *(True/False)*

8. In affine plane geometry, every point is necessarily incident with an infinite number of lines. ___________________ *(True/False)*

9. In the axiomatic approach to plane geometry, the term '*line*' is defined as an algebraic curve of degree one. ___________________ *(True/False)*

10. In the game *Set*®, cards and sets represent points and lines of an affine space. ___________________ *(True/False)*

Section B: Classical Plane Geometry

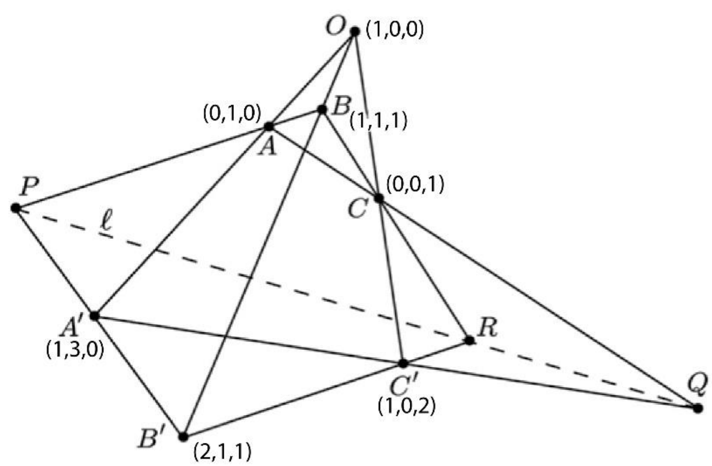
Instructions: In #11, fill in each of the blanks below using the *best* word selected from the following list. You may use a word more than once, or not at all.

plane	triangle	axioms	conic	points	distance	classical
quadrangle	concurrent	algebra	polarity	lines	angle	affine
Pappus	collinear	parallel	infinity	field	order	projective

11. *(28 points)* The classical theorem of ______________ holds in a projective plane if and only if the plane is classical, meaning that it is coordinatized by a ________________. The proof of theorem is a somewhat involved exercise using ___________________ . The theorem makes no reference to the ________________ of points on a line; nor does it require the notions of __________________ or ___________________ which are featured prominently in Euclidean plane geometry.

    We define a __________________ to be a configuration of four __________________, no three of which are __________________ . In the case of _______________ planes and _________________ planes, the existence of such a configuration is required by the ________________ , in order to exclude degenerate examples. Also for such planes, any three distinct points are either ________________ or they determine a unique _________________ .

12. *(18 points)* The figure on the right depicts a configuration of points and lines in the real projective plane. Determine the coordinates of the points P, Q, R and verify their collinearity.

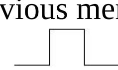


13. *(15 points)* Consider the following statement, valid in the real projective plane (also in the Euclidean plane, if one makes accommodations for parallel lines).

Theorem. *Let A, B, C be distinct points on a nondegenerate conic γ; and let a, b, c be the tangent lines to γ at the points A, B, C respectively. Consider the points $A' = b \cap c$, $B' = c \cap a$ and $C' = a \cap b$. Then the three lines AA', BB' and CC' are concurrent.*

Sketch the configuration described in this theorem. Also state the dual of this theorem, which is also a valid theorem. (*Do not prove the theorem or its dual*.)

Section C: Fractals

14. *(9 points)* Consider the curve C defined as the limit of the following infinite sequence, each member having length $\frac{5}{3}$ times the length of the previous member of the sequence. (Note that at each step, one replaces each segment ________ by ). Determine the Hausdorff dimension of C.