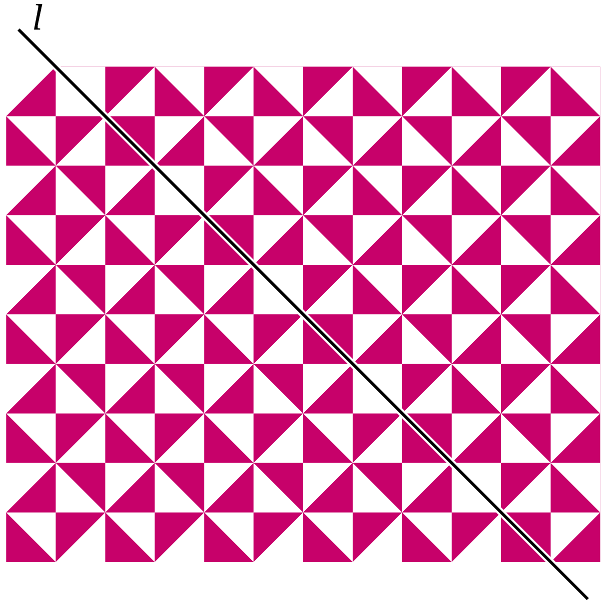
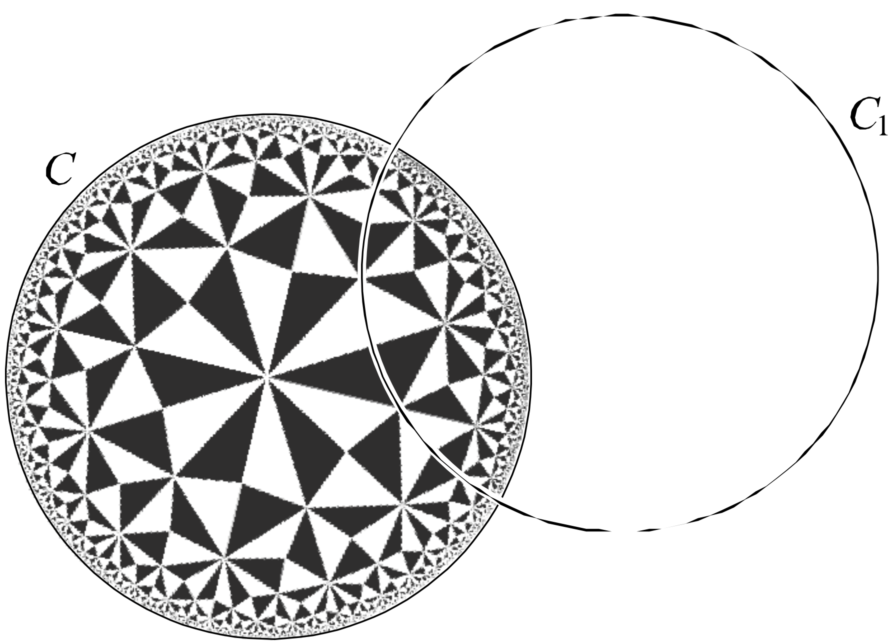
Tessellations (Regular Tilings)
in the Euclidean and Hyperbolic Planes

The figure below shows a tessellation (i.e. tiling) of the Euclidean plane with congruent (equal-sized) triangles having angles 45°-45°-90°. A reflection in the line ℓ preserves this tessellation (i.e. mirror images of triangles of the tiling, are also triangles of the tiling).



The figure below shows a tessellation of the hyperbolic plane with congruent triangles having angles 30°-45°-90° (and therefore area $\pi/12$) and a line C_1 such that reflection in C_1 preserves the tessellation. The hyperbolic plane is represented in the Poincaré model as the interior of the circle C; the line C_1 is represented by a Euclidean circle C_1 orthogonal to C; and the reflection in this line is represented by inversion in C_1 .



M.C. Escher's *Circle Limit IV* (1960) depicts a tiling of the real hyperbolic plane, seen here in the Poincaré model, by angels and demons. Any two angels are congruent in the hyperbolic plane, as are any two demons; and they are readily abstracted to give a tiling of the hyperbolic plane by congruent 45°-45°-60° triangles colored alternately black and white. Here is a copy of Escher's image, annotated by red lines that emphasize the triangular tiling: