Math 4520

Fields

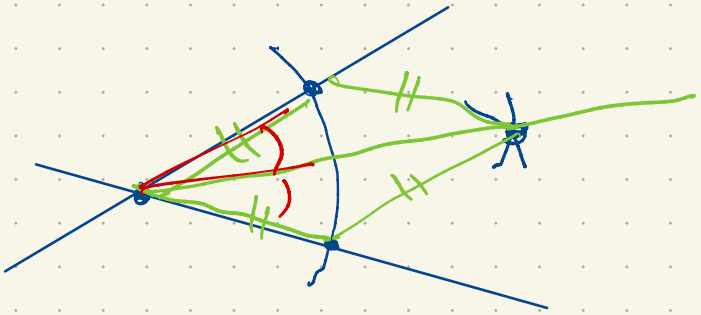
Book 3

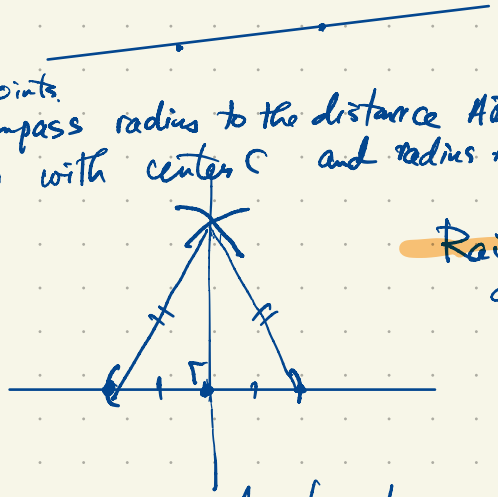
Suppose $\theta \in \mathbb{C}$ is a root of some nonzero poly $m(x) \in \mathbb{Q}[x]$.
Then take the smallest degree such $m(x)$. Assuming $m(x)$ is monic, this is the minimal polynomial of θ (over $\mathbb{Q}$). It is irreducible in $\mathbb{Q}[x]$.
Conversely, if $m(x) \in \mathbb{Q}[x]$ is a monic irreducible poly. having θ as a root, then it is the minimal poly. of θ. (over $\mathbb{Q}$).
To find $m(x)$: look at $1, \theta, \theta^2, \theta^3, \dots$ until you find a power θ^n which is a linear combination of smaller powers $1, \theta, \theta^2, \dots, \theta^{n-1}$.
$\theta^n = a_0 \cdot 1 + a_1\theta + a_2\theta^2 + \dots + a_{n-1}\theta^{n-1}$ $\iff$ θ is a root of $x^n - a_{n-1}x^{n-1} - \dots - a_1 x - a_0 \in \mathbb{Q}[x]$ (monic).

Straightedge and Compass Constructions

Here we allow certain operations:
Given two marked points on a page, we can mark the unique line through those two points.
Given any pair of points A, B we can set the compass radius to the distance AB and given a further point C, we can then mark the circle (or any arc thereof) with center C and radius AB.

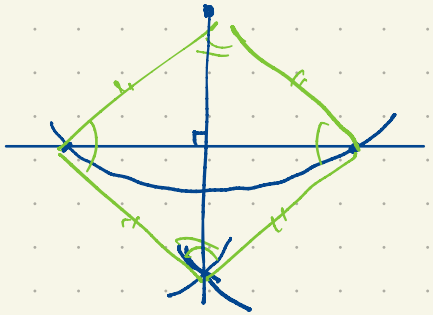
Using these tools, we can bisect any angle:



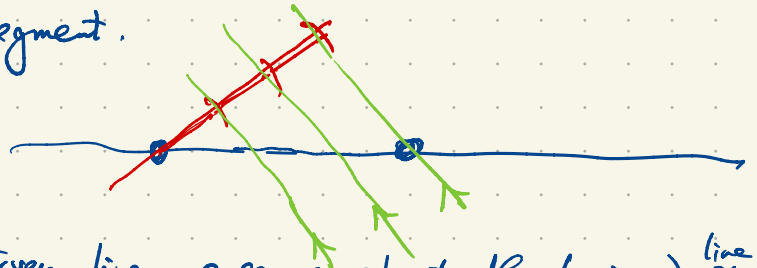


Raise a perpendicular to any line through any given point on that line

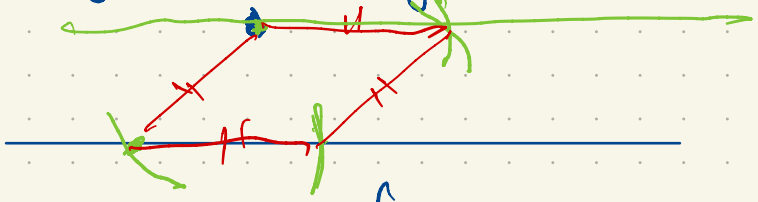
Drop a perpendicular to a line from any point not on the line:

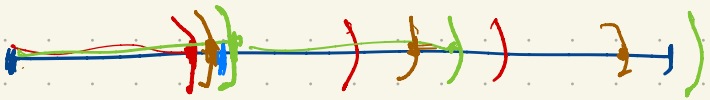


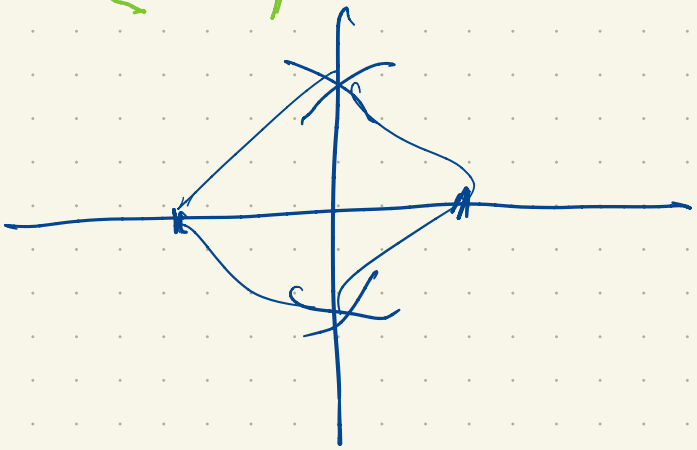
We can trisect any line segment.

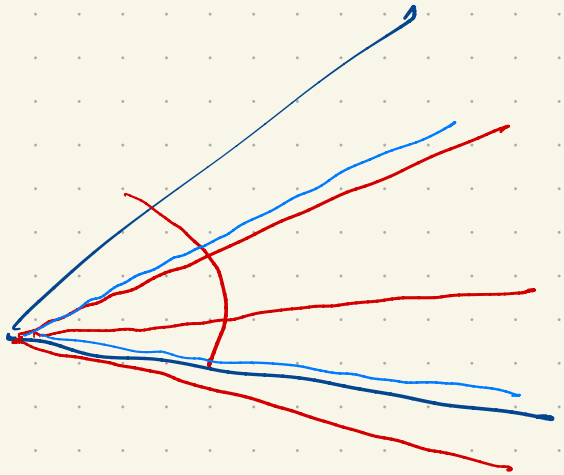


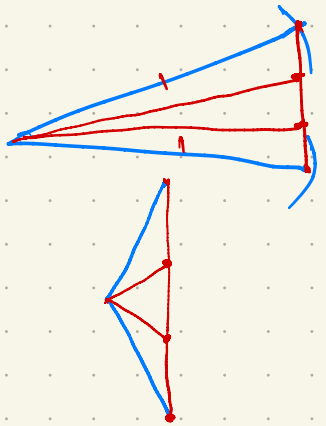
Through any point not on a given line, we can construct the (unique) line parallel to the given line.



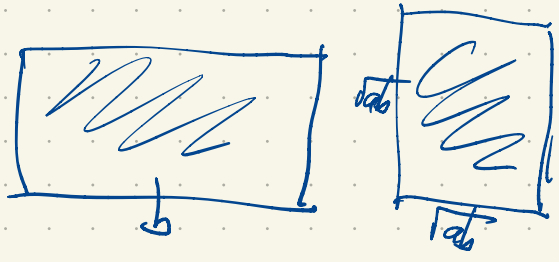








Given line segments of length a and b, we can construct a line segment of length $\sqrt{ab}$ (the geometric mean of a and b).



Given a and b,

