UNIVERSITY OF WYOMING

Math 4520—Fall 2024

Algebra III
Fields

$\mathbb{R}$ $F[\alpha] \cong F[t]/(f(t))$

$3+2\sqrt{2} = 3-2$

Department of Mathematics

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HW4

Due 5:00pm, Wednesday, December 4, 2024, on WyoCourses

Instructions: Show your work, and *check* answers whenever possible. Submit solutions through WyoCourses. See the syllabus and FAQ for general expectations regarding homework. Total value of questions: 75 points.

1. (*20 points*) Let $F = \mathbb{Q}(x)$, the field of rational functions Which of the following maps are automorphisms of the field F? For any maps that are not automorphisms, *explain why not*.

   (a) $f(x) \mapsto 5f(x) + 3$

   (b) $f(x) \mapsto f(x^2)$

   (c) $f(x) \mapsto f(x + x^3)$

   (d) $f(x) \mapsto f(7 - x)$

   (e) $f(x) \mapsto f\left(\frac{2x+5}{x+3}\right)$

2. (*20 points*) Compute the missing coefficients in the first four terms of the series expansion

   $$f(x) = \sqrt{x^2 + 3x + 4} = \boxed{} + \boxed{}\, x + \boxed{}\, x^2 + \boxed{}\, x^3 + \cdots \in \mathbb{Q}[[x]].$$

   Show your work, simplifying all coefficients in $\mathbb{Q}$ (i.e. as reduced fractions). (After finding the answer by hand, you may use the computer to check your work.)

3. (*15 points*) Consider the extension $E \supseteq F$ where $E = \mathbb{Q}(x)$ and $F = \mathbb{Q}(x^2)$.

   (a) Show that $F \cong E$.

   (b) Determine the degrees of the extensions $[E : F]$, $[E : \mathbb{Q}]$, $[F : \mathbb{Q}]$.

   (c) Find a subfield of $\mathbb{R}$ isomorphic to E.

4. (*20 points*) Consider the rational function $f(x) = \frac{1+x^2}{x^2+x^4+x^5} \in \mathbb{F}_2(x)$. Compute the missing coefficients of the first eight terms in the series expansion

   $$f(x) = \boxed{}\, x^{-2} + \boxed{}\, x^{-1} + \boxed{} + \boxed{}\, x + \boxed{}\, x^2 + \boxed{}\, x^3 + \boxed{}\, x^4 + \boxed{}\, x^5 + \cdots \in \mathbb{F}_2((x)).$$

   Show your work, simplifying all coefficients in $\mathbb{F}_2$ (i.e. as 0 or 1). (After finding the answer by hand, you may use the computer to check your work.)