HW3

Due 5:00pm, Friday, November 22, 2024, on WyoCourses

Instructions: Show your work, and *check* answers whenever possible. Submit solutions through WyoCourses. See the syllabus and FAQ for general expectations regarding homework. Total value of questions: 75 points.

1. (*20 points*) The field of order 25 may be expressed as $\mathbb{F}_{25} = \{a+b\sqrt{2} \,:\, a,b \in \mathbb{F}_5\}$. Consider the elements $\alpha = 4+\sqrt{2}$, $\beta = 3+2\sqrt{2}$ in $\mathbb{F}_{25}$. Compute each of the following, expressing your answers in the simplified form $a+b\sqrt{2}$ where $a,b \in \{0,1,2,3,4\}$.
   (a) $\alpha + \beta$
   (b) $\alpha - \beta$
   (c) $\alpha\beta$
   (d) α/β
   (e) α^3

2. (*25 points*) For the field of order 25 given in the notation of #1, find a primitive element $\gamma \in \mathbb{F}_{25}$. (Recall that this is an element satisfying $\mathbb{F}_{25} = \{0, 1, \gamma, \gamma^2, \gamma^3, \ldots, \gamma^{23}\}$.) Also, how many elements of $\mathbb{F}_{25}$ are primitive?

3. (*30 points*) Let $F = \mathbb{F}_p$ where p is prime.
   (a) Recall that there are $(p-1)p^2$ polynomials $ax^2+bx+c \in F[x]$ of degree 2, of which p^2 are monic. How many *irreducible monic* polynomials of degree 2 are there in $F[x]$? Explain.
   (b) There are $(p-1)p^3$ polynomials $ax^3+bx^2+cx+d \in F[x]$ of degree 3, of which p^3 are monic. How many *irreducible monic* polynomials of degree 3 are there in $F[x]$? Explain.