

Combinatorics

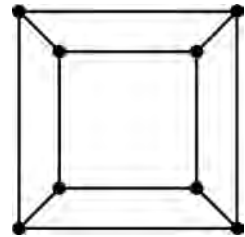
$$C(x) = \sum_{n=0}^{\infty} \binom{2n}{n} \frac{x^n}{n+1}$$

Test 1

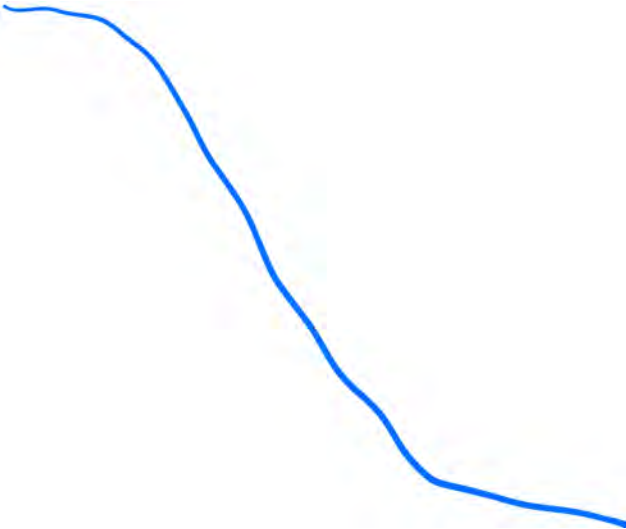
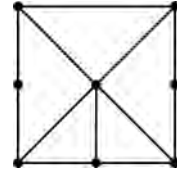
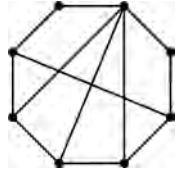
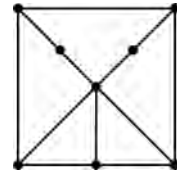
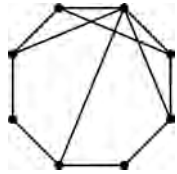
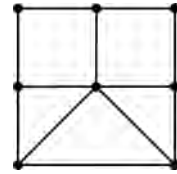
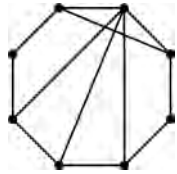
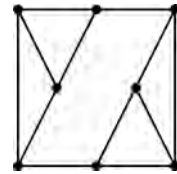
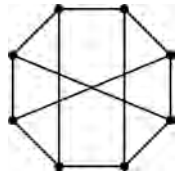
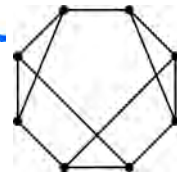
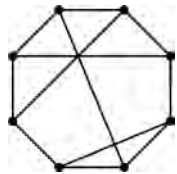
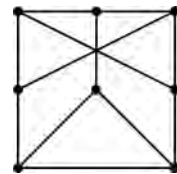
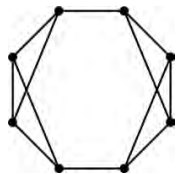
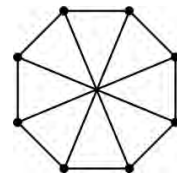
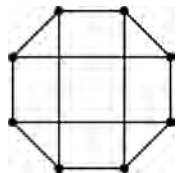
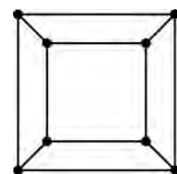
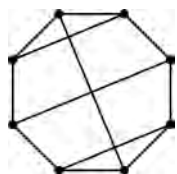
March, 2023

Instructions. The graphs considered in this test are *undirected*, with *no loops or multiple edges*. The only aids allowed are a hand-held calculator and one ‘cheat sheet’, i.e. an 8.5" × 11" sheet with information written on one side in your own handwriting. No cell phones are permitted (in particular, a cell phone may not be used as a calculator). Time permitted: 50 minutes. There are 17 bonus points available, but the maximum possible score is 100 points.

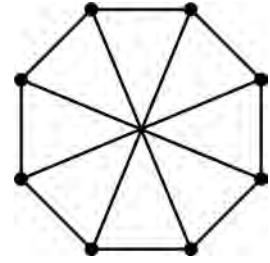
1. (16 points) Consider the planar graph Γ shown on the right. Draw the planar graph which is its dual.



2. (21 points) Several graphs of order 8, each with 12 edges, are shown. Match each graph in the left column with the one in the right column that is isomorphic to it. The first one is done for you.



3. (30 points) For the graph Γ shown on the right, determine each of the following. Explanations are optional.



(a) the diameter of Γ is

(b) the girth of Γ (i.e. the length of the shortest circuit) is

(c) the clique number $\omega(\Gamma) =$

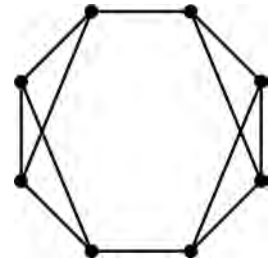
(d) the coclique number (i.e. independence number) $\alpha(\Gamma) =$

(e) the chromatic number $\chi(\Gamma) =$

(f) the number of automorphisms $|\text{Aut } \Gamma| =$

4. (20 points) Consider the graph Γ shown at the right.

(a) Is Γ planar?



(b) How many automorphisms does Γ have?

(c) What is the diameter of Γ ?

(d) What is its chromatic number $\chi(\Gamma)$?

5. (30 points) Answer TRUE or FALSE to each of the following statements.
- (a) For all $n \geq 2$, the Hamming n -cube H_n has a Hamilton circuit. _____ (True/False)
 - (b) For all $n \geq 3$, the complete graph K_n has an Euler circuit. _____ (True/False)
 - (c) There exists a graph Γ of order 7 which is isomorphic to its complement. _____ (True/False)
 - (d) There exists an infinite graph which is isomorphic to its complement. _____ (True/False)
 - (e) If a graph is connected, then its complement cannot be connected. _____ (True/False)
 - (f) There exists a graph with degree sequence $(1, 1, 2, 3, 4, 5, 6, 7)$. _____ (True/False)
 - (g) There exists a graph with degree sequence $(0, 1, 2, 3, 4, 5, 6, 7)$. _____ (True/False)
 - (h) The Petersen graph has more automorphisms than any other graph of order 10. _____ (True/False)
 - (i) There exists a graph Γ of order 6 such that neither Γ nor its complement $\bar{\Gamma}$ contains a triangle. _____ (True/False)
 - (j) Every connected graph with 7 vertices and 6 edges is planar. _____ (True/False)