

HW4

(Due 5:00 pm, Wednesday, May 3, 2023 on *WyoCourses*)

Instructions: See the syllabus for general instructions for completing homework. Further details are found at the FAQ page linked from the syllabus. Always check your answers wherever feasible. Write clearly, using complete sentences where appropriate, and always using correct notation. *When the answer is an integer, give its exact value, explicitly and without approximation.*

- (10 points) I have 5 different used books written by my favorite author. How many ways can I distribute them to 9 students
 - if no student receives more than one book?
 - without limiting the number of different books each student receives?

- (10 points) I have 5 identical silver coins. How many ways can I distribute them to 9 students
 - if no student receives more than one coin?
 - without limiting the number of different coins each student receives?

- (15 points) Denote $F(x) = \frac{1}{\sqrt{1-4x}}$.
 - Determine $\int_0^x F(t) dt$.
 - In class on March 12 (and in an accompanying handout on Catalan numbers), we proved the identity

$$F(x) = \sum_{n=0}^{\infty} \binom{2n}{n} x^n.$$

Integrate term by term to find a series expansion for the integral in (a).

- Use this to give an alternative proof of the formula for the generating function of $\frac{1}{n+1} \binom{2n}{n}$.
- (20 points) Denote the standard n -set as usual by $[n] = \{1, 2, 3, \dots, n\}$. Count the number of surjections $f : [n] \rightarrow [k]$ (i.e. functions that are onto)
 - if $k = n$;
 - if $k = n-1$;
 - if $k = n-2$;
 - if $k = 2$.

5. (15 points) Given a sequence a_0, a_1, a_2, \dots , denote its generating function by $A(x) = \sum_{n=0}^{\infty} a_n x^n$. Also define its sequence of *forward differences* by $\Delta a_0, \Delta a_1, \Delta a_2, \dots$ where $\Delta a_n = a_{n+1} - a_n$.
- (a) Denote by $D(x)$ the generating function of the new sequence Δa_n . Express $D(x)$ algebraically in terms of $A(x)$.
 - (b) Suppose that M&M's are manufactured in 8 colors, and that a_n counts the number of possible outcomes when selecting a handful of n M&M's at random. Express the generating function $A(x)$ (simplified and in closed form).
 - (c) Determine the generating function $D(x)$ for the sequence Δa_n in the situation described in (b). Express $D(x)$ in simplified form.
6. (10 points) At the end of a semester, a teacher assigns letter grades A,B,C,D,F to each of her 22 students.
- (a) How many outcomes are possible in all?
 - (b) In how many ways may the teacher assign five A's, seven B's, seven C's, two D's and one F?