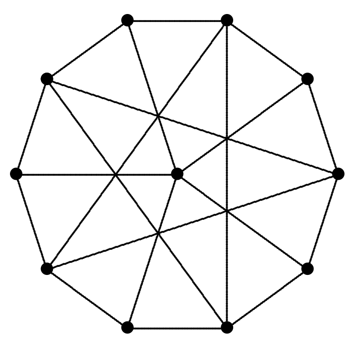
University of Wyoming — Math 3700—Spring 2023 — Combinatorics — Department of Mathematics

HW2

(Due 5:00 pm, Friday, March 3, 2023 *on WyoCourses*)

Instructions: See the syllabus for general instructions for completing homework. Further details are found at the FAQ page linked from the syllabus. Always check your answers wherever feasible. Write clearly, using complete sentences where appropriate, and always using correct notation. All graphs here are simple (undirected, with no loops or multiple edges).

1. *(20 points)* A small private school has eleven classes, represented by the vertices of the following graph.



If two classes have at least one student in common, we draw an edge between the corresponding vertices. If two classes have no students in common, the corresponding vertices are unjoined. At the end of the semester, the school must schedule eleven two-hour final exams, one for each class. The school wants to run different exams simultaneously, in order to finish all the exams as quickly as possible; but they cannot schedule two exams at the same time if there are any students in both classes.

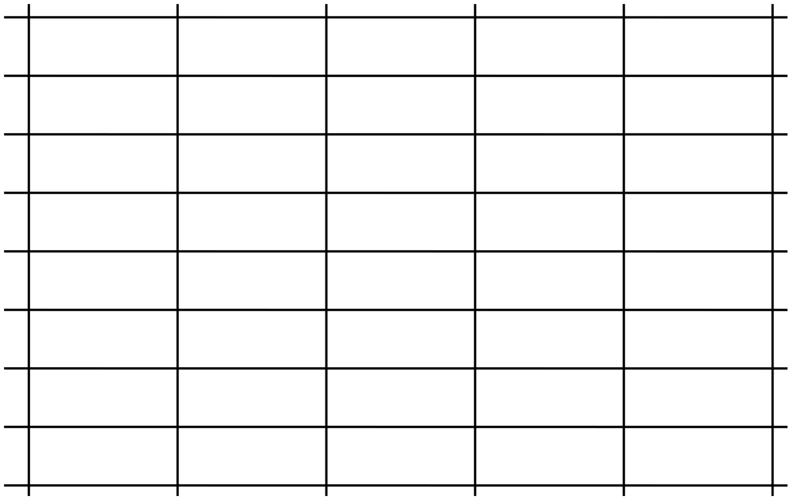
   (a) What is the maximum number of classes whose exams can be conducted simultaneously without any conflicts?

   (b) What is the minimum number of two-hour time slots required to schedule all exams without any conflicts?

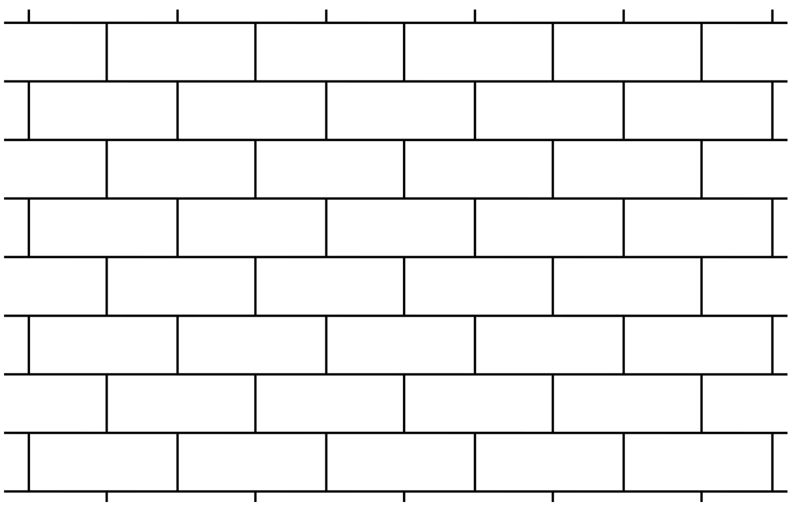
2. *(20 points)* For each of the following planar maps, determine the smallest number of colors with which the regions may be properly colored. (A proper coloring of regions requires you to use different colors for any two regions sharing a boundary edge. This restriction does not apply for regions sharing a single boundary point such as a corner. And note that we are coloring regions here, not vertices.) In each case, the

map continues beyond what is shown, but you can safely ignore partial regions on the edges of the maps, without altering the number of colors required.

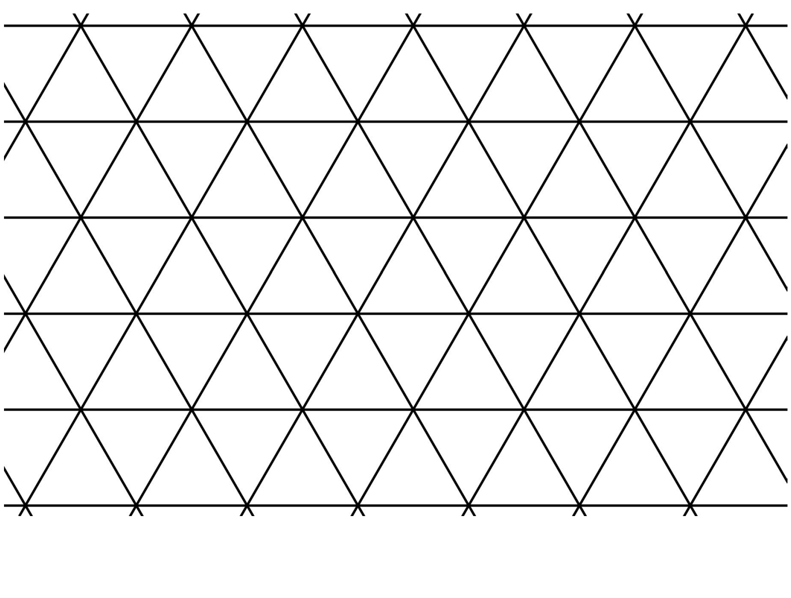
(a)

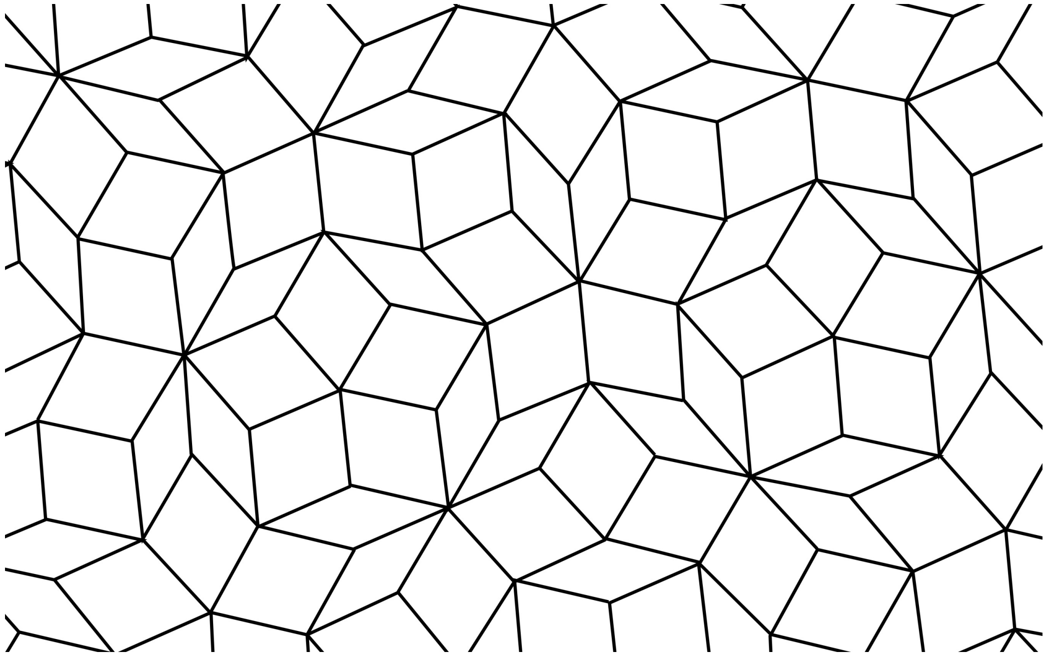


(b)



(c)



(d) 

Hint: In (d), start by coloring one of the inside regions, then gradually work your way outwards in 'rings' around you starting region, trying to use as few colors as possible.

3. *(25 points)* Let P be the Petersen graph. Construct a graph P' with 15 vertices and 30 edges as follows: Vertices of P' correspond to the edges of P. Two vertices of P' are adjacent whenever the corresponding edges in P share a vertex. Of course P' is 4-regular since every edge of P touches 4 other edges.

   (a) Draw a picture of P'. You should try to make your picture as clear as possible, preferably revealing as much symmetry as seen in the illustration of P.

   (b) What is the diameter of P'?

   (c) What is the clique number $\omega(P')$?

   (d) What is the coclique number (i.e. independence number) $\alpha(P')$?

   (e) What is the chromatic number $\chi(P')$?

   Hint: It is possible to rephrase a problem about vertices of P', as an equivalent problem about edges of P. For example, what is the largest set of edges you can find in P, no two of which share a vertex? This will answer one part of #3.