Algebra I

Group Theory

Book 1

A group is a set G with a binary operation $*$ which has an identity element; the operation is associative; and every element has an inverse.

Eg. $\mathbb{R}$ = set of real numbers under addition '+'. Its identity element is 0.

$$\left.\begin{aligned} 0 + x &= x \\ (x+y)+z &= x+(y+z) \\ x + (-x) &= 0 = (-x) + x \end{aligned}\right\} \quad \text{for all } x, y, z \in \mathbb{R}$$

$(\mathbb{R}, +)$ is a group.

$(\mathbb{R}, \times)$ (real numbers under multiplication is almost but not quite a group. (0 does not have an inverse). 1 is the identity.

$\mathbb{R}^\times$ = {all nonzero real numbers} = $\{a \in \mathbb{R} : a \neq 0\}$ is a group under multiplication.

$$\begin{aligned} 1a &= a \\ (ab)c &= a(bc) \\ a \cdot a^{-1} &= a^{-1} \cdot a = 1 \qquad a^{-1} = \frac{1}{a} \end{aligned} \qquad \text{for all } a, b, c \in \mathbb{R}^\times.$$