HW2

(Due 5:00 pm, Thursday, October 26, 2023 *on WyoCourses*)

Instructions: See the syllabus for general instructions for completing homework. Further details are found at the FAQ page linked from the syllabus. Always check your answers wherever feasible. Write clearly, using correct notation.

1. *(10 points)* Complete the following Cayley table for a group $G = \{u, v, w, x, y\}$ of order 5:

	u	v	w	x	y
u			w		
v		x			
w			v		
x					
y					

2. *(10 points)* Let $\mathbb{F}_3 = \{0, 1, 2\}$ be the field of order 3 (with addition and multiplication mod 3). Let G be the group of upper triangular matrices of the form

$$\begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}$$

where $a, b, c \in \mathbb{F}_3$, so that $|G| = 27$. How many elements of each order does G have?

3. *(10 points)* Let $G = SL_2(\mathbb{F}_3)$, the group of all 2×2 matrices over $\mathbb{F}_3$ with determinant equal to 1, so that $|G| = 24$. How many elements of each order does G have?

In class we talked about the product of all the elements in a finite abelian group G, denoting this element π in one of our proofs.

4. *(10 points)* Show that 'the product of all elements in G' is not well-defined in the case of a finite nonabelian group G. Specifically, show that the product can differ depending on the order in which one takes the product.

5. *(20 points)* Let G be a finite abelian group, and let π be the product of the elements of G.
   (a) Show that π has order 1 or 2.
   (b) Give an example of a group G of even order in which π is the identity.
   (c) Give an example of a group G of even order in which π has order 2.
   (d) Show that π has order 2 iff G has a *unique* element of order 2.