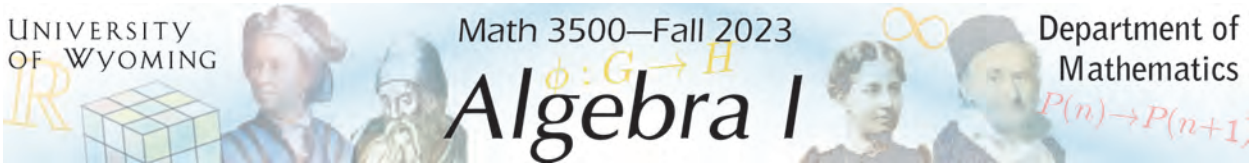

HW1

(Due 5:00 pm, Friday, September 22, 2023 *on WyoCourses*)

Instructions: See the syllabus for general instructions for completing homework. Further details are found at the FAQ page linked from the syllabus. Always check your answers wherever feasible. Write clearly, using correct notation.

1. *(28 points)* Each of the following objects (in 2 or 3 dimensions) has a symmetry group G. In each case, determine the order $|G|$; also determine whether or not G is abelian (e.g. abelian of order 3, infinite nonabelian, etc.).
   (a) a regular pentagonal prism (whose cross sections are regular pentagons) 
   (b) the letter E
   (c) the string of letters S O S
   (d) the infinite string of letters $\cdots$ E E E E E E E E E E $\cdots$
   (e) the string of letters Y O U
   (f) the infinite string of letters $\cdots$ H H H H H H H H H H $\cdots$
   (g) a brick of size $2'' \times 3'' \times 8''$ 

2. *(26 points)* (a) Find the four missing elements in the following subgroup of S_8:

   $\{\ (),\ (1324)(5768),\ (1423)(5867),\ (1526)(3847),$ ____, ____, ____, ____ $\}$.

   (b) How many elements of each order does this group have?
   (c) Is this group abelian?

3. *(10 points)* In the group S_6, find an element that commutes with $(123)(456)$ but not with $(14)(25)(36)$.

4. *(16 points)* (a) Find the three missing elements in the following subgroup of $GL_2(\mathbb{R})$:

   $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}, \right.$ ____, ____, ____ $\left. \right\}$.

   (b) How many elements of each order does this group have?
   (c) Is this group abelian?