University of Wyoming — Math 3205—Fall 2020 — **Analysis I** — Department of Mathematics

HW1

Due 5pm Friday, September 18, 2020

Instructions: Submit your work online through WyoCourses. Typical choices of format for submission:

- Handwritten work (from paper), scanned and submitted as a pdf document. Camera photos may be acceptable but a scanner app is strongly preferred as this will help to control shadows, improve document readability, control borders of pages, etc.
- Handwritten work (using stylus on a tablet using hand notetaking software such as Notability, Evernote, Goodnotes, etc.) exported as pdf.
- Document in LaTeX, Word or other software, exported as pdf.

Other formats will be accepted but if the images are unclear, this may delay the grading process. Please contact me if you need help with submitting documents. Text input will only be used as a last resort because this makes it impossible to use standard mathematical notation. *See also the syllabus (and the link to FAQ's) regarding general expectations regarding homework*.

1. (20 points; from the second instructional video, Decimals)
   When discussing decimal expansions, we say that the real number 1 has exactly two decimal expansions. Here we regard 1 the same as 1.0000 and 1.000000… (the point is that all the digits are the same here, whether or not they are explicitly *shown*). Decimal expansions in other bases are defined exactly like base 10, as limits of infinite series.
   (a) Find the unique rational number whose ternary (i.e. base 3) expansion is 1012.102102102… .
   (b) Find a real number having a unique decimal expansion but two distinct ternary expansions.
   (c) Find a real number having a unique ternary expansion but two distinct decimal expansions.
   (d) Find a real number having two distinct decimal expansions and two distinct ternary expansions.
   (e) Find a real number having a unique decimal expansion and a unique ternary expansion.

2. (20 points) Show from the definition that $\lim_{x\to 3} x^2 = 9$. Your proof must be written in complete sentences beginning with "Let $\varepsilon > 0$." You will need to find an appropriate δ (which must depend on ε) which satisfies the definition of a limit.

Use the following example as a model for your proof:

Theorem: $\lim_{x\to 3} \frac{x}{x+1} = \frac{3}{4}$.

Proof: Let $\varepsilon > 0$. Denote $(x) = \frac{x}{x+1}$. We consider two cases.

- First suppose ≥ 1. In this case, it suffices to take $\delta = 1$. Whenever $0 < |x - 3| < 1$ we have $2 < x < 4$, so
$$\frac{2}{5} < \frac{x}{x+1} < \frac{4}{3}$$
and $\left|f(x) - \frac{3}{4}\right| < \frac{7}{12} < \varepsilon$ as required.
- Otherwise $0 < \varepsilon < 1$. In this case, we may take $\delta = \varepsilon$. Whenever $0 < |x - 3| < \delta$ we have $3 - \varepsilon < x < 3 + \varepsilon$. Noting that both endpoints of this interval are positive, we have
$$\frac{3-\varepsilon}{4+\varepsilon} < f(x) < \frac{3+\varepsilon}{4-\varepsilon}$$
and so
$$-\varepsilon < -\frac{7\varepsilon}{12} < -\frac{7\varepsilon}{4(4+\varepsilon)} < f(x) - \frac{3}{4} < \frac{7\varepsilon}{4(4-\varepsilon)} < \frac{7\varepsilon}{12} < \varepsilon\,.$$

In both cases $\left|f(x) - \frac{3}{4}\right| < \varepsilon$. □

3. (20 points) Consider the function $f(x) = \begin{cases} \frac{\sin x}{x}, & \text{if } x \neq 0; \\ 1, & \text{if } x = 0. \end{cases}$

(i) Find $f^{(n)}(x)$ for $n = 0,1,2$. (Use the definition of derivative whenever this is required.)

(ii) Write down the Taylor polynomial of degree 2 for f.