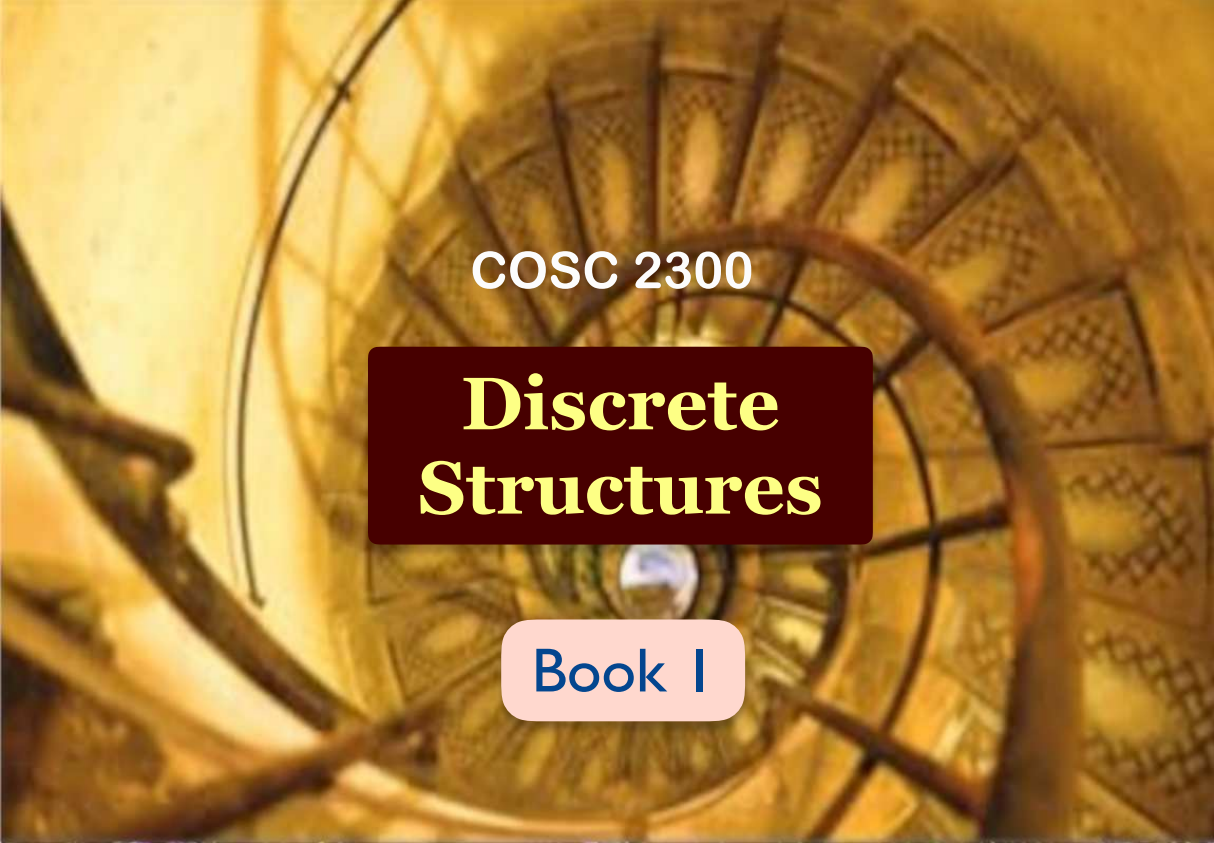
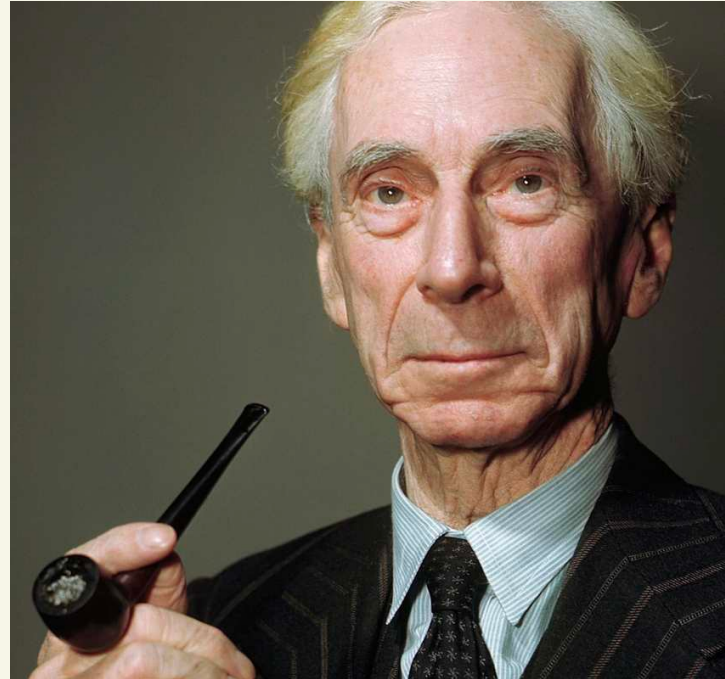




COSC 2300

Discrete Structures

Book I



Bertrand Russell (1872-1970)

In a particular village there is a gentleman barber who shaves every gentleman who does not shave himself.

Does the barber shave himself?

No true/false answer can be given that is consistent.
This is an example of Russell's Paradox.

Other forms:

P: The statement P (ie. this statement) is false.

Avoiding self-referential statements:

A: The statement B is true.

B: The statement A is false.

Propositional Logic (Propositional Calculus)

R: Bertrand Russell was born in 1872. (True)

M: Mars is the second planet from the sun. (False)

J: January is the first month of the calendar year. (True)

T: A triangle has four sides. (False)

Syntactic vs. Semantic features language
syntax (symbolic) meaning (semantics)
true/false

R: Bertrand Russell was born in 1872.

(True)

M: Mars is the second planet from the sun.

(False)

J: January is the first month of the calendar year.

(True)

T: A triangle has four sides. (False)

"Exclusive or" of T and T is F. Avoid!

$R \vee M$ is True

↑
"or"
"join"
"disjunction"

$R \wedge M$ is False

↑
"and"
"meet"
"conjunction"

Truth Table: for two statements P, Q
(atomic propositions)

P	$\neg P$
T	F
F	T

Conditional Statements $P \rightarrow Q$ ("P implies Q")
(implication) ("If P then Q")

$R \rightarrow M$ is false
 $J \rightarrow R$ is true.
 $T \rightarrow M$ is true.

$(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$
↑
equivalence of propositions
 $P \wedge Q \equiv Q \wedge P$

$P \rightarrow Q \equiv (\neg P) \vee Q$

P	Q	$P \rightarrow Q$	$\neg P$	$(\neg P) \vee Q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

"inclusive" or

P	Q	$P \vee Q$	$P \wedge Q$
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	F

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

$$(P \vee Q) \vee R \equiv P \vee (Q \vee R)$$

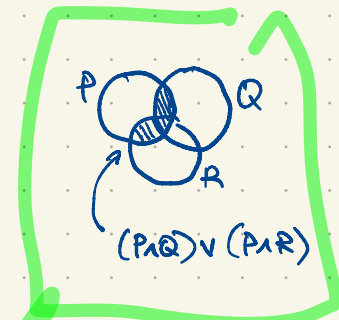
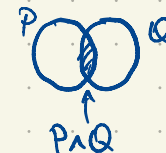
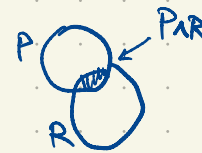
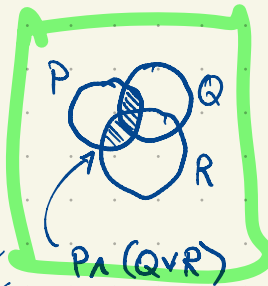
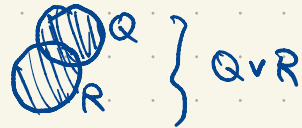
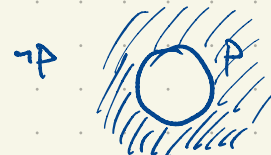
$$a(b+c) = ab+ac$$

$$a+bc \neq (a+b)(a+c)$$

$$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$$

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$$

This can be verified using a truth table (with eight rows)
or using Venn diagram



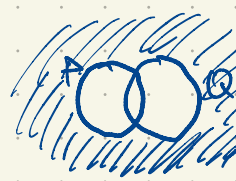
$$\neg(P \wedge Q) \equiv (\neg P) \vee (\neg Q)$$

"P and Q are not both true, i.e. at least one of 'P and Q' fails"



$$\neg(P \vee Q) \equiv (\neg P) \wedge (\neg Q)$$

"Neither P nor Q is true; both are false"



De Morgan's Rules

$$\neg(P_1 \wedge P_2 \wedge P_3 \wedge \dots \wedge P_k) \equiv (\neg P_1) \vee (\neg P_2) \vee \dots \vee (\neg P_k)$$

P_i : the i th student is wearing red
 $P_1 \wedge P_2 \wedge \dots \wedge P_k$: all k students are wearing red
 $\neg(P_1 \wedge P_2 \wedge \dots \wedge P_k)$: not all k students are wearing red
 i.e. at least one of the k students

$$\neg(P_1 \vee P_2 \vee \dots \vee P_k) \equiv (\neg P_1) \wedge (\neg P_2) \wedge \dots \wedge (\neg P_k)$$

is not wearing red.
 none of the k students are wearing red.

"All the students are not wearing red." Ambiguous! Which of the above does the person mean?

$$P \vee (\neg P)$$

P	$\neg P$	$P \vee (\neg P)$
T	F	T
F	T	T



$P \vee (\neg P)$ is a tautology: its truth value is always true regardless of P.

P: "There exist integers x, y, z such that $x^3 + y^3 + z^3 = 42$."

$P \vee (\neg P)$ says that P is either true or false.

$P \wedge (\neg P)$ is a contradiction

P	$\neg P$	$P \wedge (\neg P)$
T	F	F
F	T	F

$P \wedge (Q \vee R)$ is neither a tautology nor a contradiction: it depends on P, Q, R (it is a contingent proposition).

$P \rightarrow (Q \wedge (\neg Q))$ cannot be true unless P is false.

$$P \rightarrow (Q \wedge (\neg Q)) \equiv \neg P$$

$P \rightarrow (Q \vee (\neg Q))$ is a tautology.

$P \rightarrow Q$ Conditional statement "If P then Q" or "P implies Q" "Q whenever P"	P	Q	$P \rightarrow Q$	Biconditional statement $P \leftrightarrow Q$ $P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$	P	Q	$P \leftrightarrow Q$
	T	T	T		T	T	T
	T	F	F		T	F	F
	F	T	T		F	T	F
	F	F	T		F	F	T

"x is even" $\leftrightarrow$ " x^2 is even"
"P if and only if Q"
"P iff Q"

This is a true statement for integers.
The conditional statement
 $Q \rightarrow P$
is the converse of $P \rightarrow Q$.

In order to prove $P \rightarrow Q$, one typically does so by assuming P and then proceeding to prove Q as a consequence.

In order to prove $P \leftrightarrow Q$, one typically proves $P \rightarrow Q$ and then also prove $Q \rightarrow P$.

Sets e.g. the set $A = \{1, 2, 5\}$ contains three ^(members) elements 1, 2, 5:

Lin

$$2 \in A$$

ε epsilon

$$1 \in A$$

$$2 \in A$$

$$5 \in A$$

$$7 \notin A \equiv \neg (7 \in A)$$

$$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$$

$$\{\} = \emptyset$$

the empty set

nothing

$B = \{1, 4, \{1, 7\}, \{1, 2, 4\}, \{2\}\}$ has 5 elements

$$1 \in B$$

$$2 \notin B$$

$$\{2\} \in B$$

$$7 \notin B$$

$$13 \notin B$$

$$1 \in \{1, 7\} \in B$$

$$\{1, 1, 3\} = \{1, 3\} \\ = \{3, 1\}$$

A set is completely specified by its elements.

Discrete vs. Continuous

$$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\} \quad \mathbb{R} \quad [5, 8] = \{x \in \mathbb{R} \mid 5 \leq x \leq 8\}$$



If A and B are sets then their ⁵union is $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

$A \subseteq B$ means $x \in B$ for all $x \in A$. (Every element of A is an element of B .) ⁸intersection: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

" A is a subset of B "

If $A = B$ then $A \subseteq B$. (Not conversely!)

$A = B$ is equivalent to $A \subseteq B$ and $B \subseteq A$.

$$\subsetneq \subsetneq \subsetneq \bigcirc_A^B$$

inclusive



Warning: $A \subset B$ can mean " A is a proper subset of B " but some people use it to just mean " A is a subset of B ".

(A is a subset of B but not equal to B) Is there any confusion between $x < y$ and $x \leq y$?

$$\{\} = \emptyset$$

The power set of a set A is the set of all subsets of A :

$$\mathcal{P}(A) = \{B \mid B \subseteq A\}$$

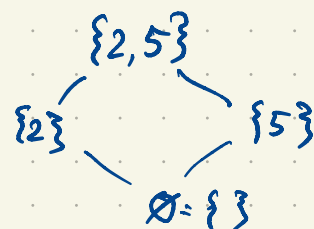
$$\mathcal{P}(\{2, 5\}) = \{\emptyset, \{2\}, \{5\}, \{2, 5\}\}$$

$$\{2\} \subseteq \{2, 5\}$$

$$\{2\} \not\subseteq \{2, 5\}$$

$$2 \in \{2, 5\}$$

$$\{2\} \in \mathcal{P}(\{2, 5\})$$



Can a set be an element of itself?

Let $S = \{x \mid x \text{ is a set, } x \notin x\}$

Does $S \in S$? If $S \in S$ then $S \notin S$.
If $S \notin S$ then $S \in S$.

