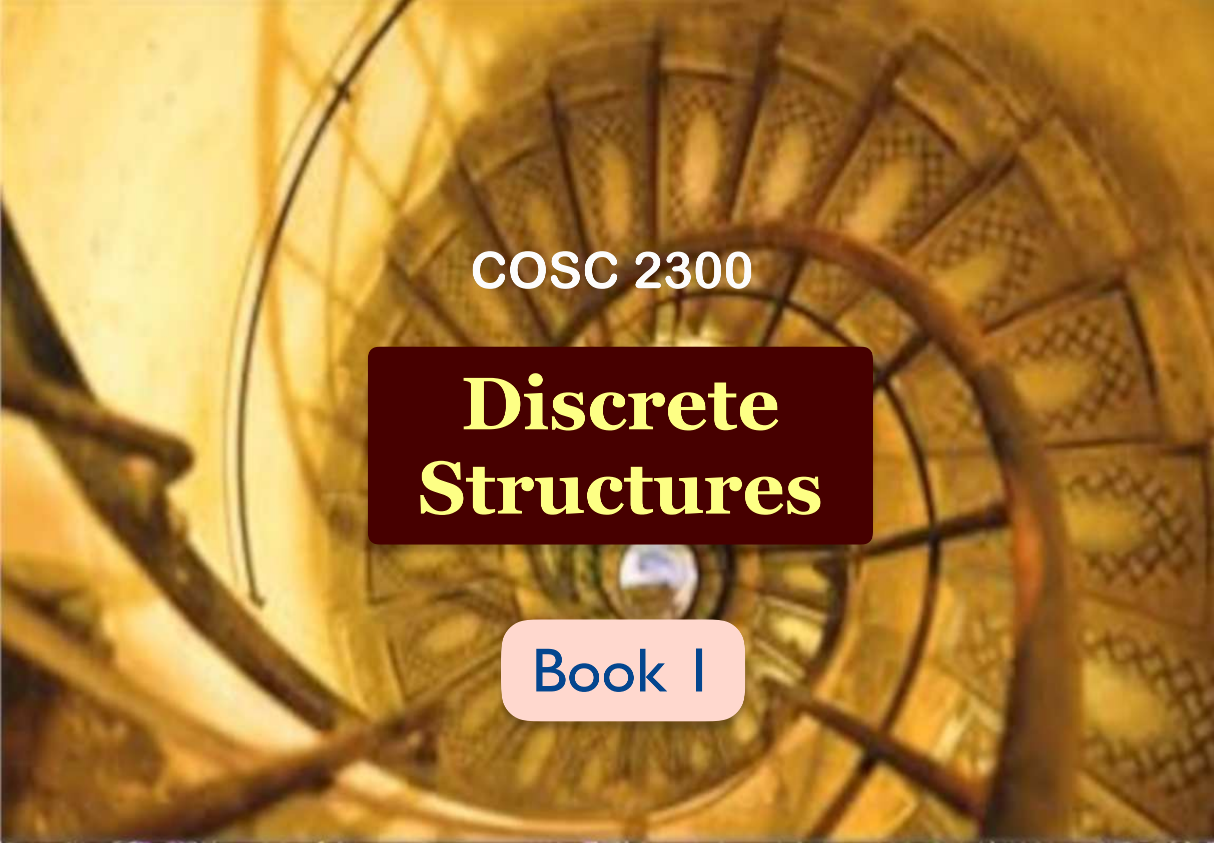
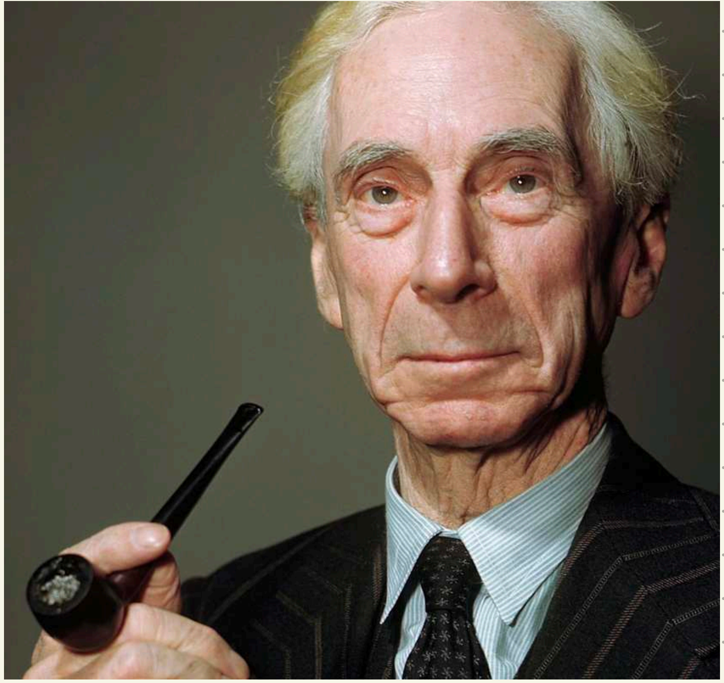


COSC 2300

Discrete Structures

Book 1



Bertrand Russell (1872-1970)

In a particular village there is a gentleman barber who shaves every gentleman who does not shave himself.

Does the barber shave himself?

No true/false answer can be given that is consistent.

This is an example of Russell's Paradox.

Other forms:

P: The statement P (ie. this statement) is false.

Avoiding self-referential statements:

A: The statement B is true.

B: The statement A is false.

Propositional Logic (Propositional Calculus)

R: Bertrand Russell was born in 1872. (True)

M: Mars is the second planet from the sun. (False)

J: January is the first month of the calendar year. (True)

T: A triangle has four sides. (False)

Syntactic vs. semantic features language

syntax (symbolic)

meaning (semantics) true/false

R: Bertrand Russell was born in 1872. (True)
M: Mars is the second planet from the sun. (False)
J: January is the first month of the calendar year. (True)
T: A triangle has four sides. (False)

"Exclusive or" of T and T is F. Avoid!

$R \vee M$ is True

"or"
"join"
"disjunction"

$R \wedge M$ is False

"and"
"meet"
"conjunction"

Truth Table : for two staments P, Q
(atomic propositions)

"inclusive" or

P	Q	$P \vee Q$	$P \wedge Q$
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	F

P	$\neg P$
T	F
F	T

Conditional Statements $P \to Q$ ("P implies Q")
(implication) ("If P then Q")

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

$R \to M$ is false
$J \to R$ is true.
$T \to M$ is true.

$(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$

equivalence of propositions

$P \wedge Q \equiv Q \wedge P$

$P \to Q \equiv (\neg P) \vee Q$

P	Q	$P \to Q$	$\neg P$	$(\neg P) \vee Q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$(P \vee Q) \vee R \equiv P \vee (Q \vee R)$

$a(b+c) = ab + ac$

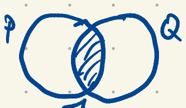
$a + bc \neq (a+b)(a+c)$

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$

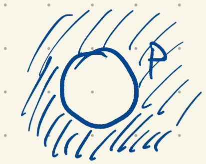
This can be verified using a truth table (with eight rows) or using Venn diagram

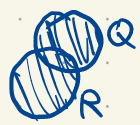
$P \wedge Q$



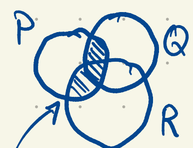
$P \vee Q$

$\neg P$

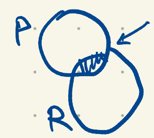




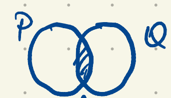
$\}\ Q \vee R$



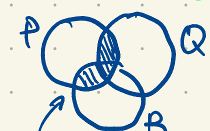
$P \wedge (Q \vee R)$



$P \wedge R$



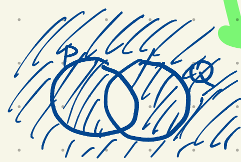
$P \wedge Q$



$(P \wedge Q) \vee (P \wedge R)$

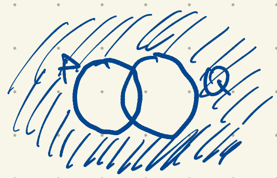
$\neg(P \wedge Q) \equiv (\neg P) \vee (\neg Q)$

"P and Q are not both true, i.e. at least one of P and Q fails"



$\neg(P \vee Q) \equiv (\neg P) \wedge (\neg Q)$

"Neither P nor Q is true; both are false".



De Morgan's Rules

$\neg(P_1 \wedge P_2 \wedge P_3 \wedge \cdots \wedge P_k) \equiv (\neg P_1) \vee (\neg P_2) \vee \cdots \vee (\neg P_k)$

P_i : the i^{th} student is wearing red

$P_1 \wedge P_2 \wedge \cdots \wedge P_k$: all k students are wearing red

$\neg(P_1 \wedge P_2 \wedge \cdots \wedge P_k)$: not all k students are wearing red i.e. at least one of the k students is not wearing red.

$\neg(P_1 \vee P_2 \vee \cdots \vee P_k) \equiv (\neg P_1) \wedge (\neg P_2) \wedge \cdots \wedge (\neg P_k)$: none of the k students are wearing red.

"All the students are not wearing red." Ambiguous! which of the above does the person mean?

$P \vee (\neg P)$

P	$\neg P$	$P \vee (\neg P)$
T	F	T
F	T	T

$P \vee (\neg P)$ is a tautology: its truth value is always true regardless of P.

P: "There exist integers x, y, z such that $x^3 + y^3 + z^3 = 42$."

$P \vee (\neg P)$ says that P is either true or false.

$P \wedge (\neg P)$ is a contradiction

P	$\neg P$	$P \wedge (\neg P)$
T	F	F
F	T	F

$P \wedge (Q \vee R)$ is neither a tautology nor a contradiction: it depends on P, Q, R (it is a contingent proposition).

$P \to (Q \wedge (\neg Q))$ cannot be true unless P is false.

$P \to (Q \wedge (\neg Q)) \equiv \neg P$.

$P \to (Q \vee (\neg Q))$ is a tautology.

$P \to Q$

Conditional statement

"If P then Q"

or "P implies Q"

"Q whenever P"

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional statement $P \leftrightarrow Q$

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

$P \leftrightarrow Q \equiv (P \to Q) \wedge (Q \to P)$.

"x is even" $\leftrightarrow$ "x^2 is even"

This is a true statement for integers.

"P if and only if Q"

"P iff Q"

The conditional statement $Q \to P$ is the converse of $P \to Q$.

In order to prove $P \to Q$, one typically does so by assuming P and then proceeding to prove Q as a consequence.

In order to prove $P \leftrightarrow Q$, one typically proves $P \to Q$ and then also prove $Q \to P$.

Sets e.g. the set $A = \{1, 2, 5\}$ contains three elements (members) $1, 2, 5$:

$1 \in A$
$2 \in A$
$5 \in A$
$7 \notin A \equiv \neg (7 \in A)$

\in

$2 \in A$

ε epsilon

$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, 4, \cdots\}$

$\{\ \} = \varnothing$

the empty set

\varnothing

$B = \{1, 4, \{1,7\}, \{1,2,4\}, \{2\}\}$ has 5 elements

$1 \in B$
$2 \notin B$
$\{2\} \in B$
$7 \notin B$
$13 \notin B$
$1 \in \{1,7\} \in B$.