

Name *Solution Key*

UNIVERSITY
OF WYOMING

COSC 2300—Fall 2026

Computer Science

Discrete Structures

Quiz 4

Thursday, October 22, 2026

A sequence of pairs of numbers $(1, 0)$, $(2, 1)$, $(7, 4)$, ... is defined recursively by

$$(a_0, b_0) = (1, 0); \quad \text{and} \\ (a_{n+1}, b_{n+1}) = (2a_n + 3b_n, a_n + 2b_n) \quad \text{for all } n \geq 0.$$

Compute the next three pairs of numbers (a_n, b_n) in the sequence; and complete the missing entries in the table below.

n	a_n	b_n	$a_n^2 - 3b_n^2$
0	1	0	1
1	2	1	1
2	7	4	1
3	26	15	1
4	97	56	1
5	362	209	1

In the last column, enter the missing values of $a_n^2 - 3b_n^2$ for $n = 3, 4, 5$.

Remarks: All pairs (a_n, b_n) generated by this recursion are solutions of $x^2 - 3y^2 = 1$. This follows induction starting with $(1, 0)$ as the initial case. Assuming (a_n, b_n) is a solution, then

$$a_{n+1}^2 - 3b_{n+1}^2 = (2a_n + 3b_n)^2 - 3(a_n + 2b_n)^2 = a_n^2 - 3b_n^2 = 1$$

So (a_{n+1}, b_{n+1}) is also a solution. It follows that the equation $x^2 - 3y^2 = 1$ has infinitely many solutions. Moreover, we can prove that all solutions of $x^2 - 3y^2 = 1$ have the form $(\pm a_n, \pm b_n)$ where (a_n, b_n) is in the sequence generated above.

You were not required to prove these facts. My only hope was that students would notice this pattern and use it to detect any incorrect pairs (a_n, b_n) in their sequence.