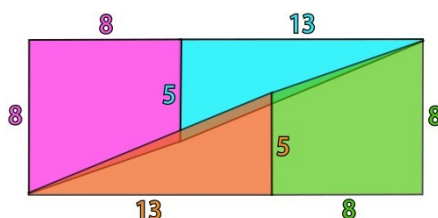


Discrete Structures

Solutions to HW3

1. The four colored pieces (formed by decomposing the 13×13 square) overlap slightly when used to cover the 8×21 rectangle. A picture which exaggerates this discrepancy is shown here:



The reason for the mismatch is that the sloping sides of the triangles is $\frac{5}{13} \approx 0.385$ — slightly steeper than the sloping sides of the trapezoids, which have slope $\frac{5}{8} = 0.375$. The actual area of the overlapping portion is one square unit, which is entirely within the black outlines of the four pieces.

2. The statement $5 = -3$ does imply that $16 = 16$ (as printed) but the converse is not valid. Never confuse a conditional statement with its converse.
3. The argument ignores the fact that integer values for x, y, z may be negative. In fact, at least one integer solution

$$42 = (-80\,538\,738\,812\,075\,974)^3 + 80\,435\,758\,145\,817\,515^3 + 12\,602\,123\,297\,335\,631^3$$

is known.

4. The error presumably arises from confusing an equation $ax^2+bx+c=0$ (which has at most two solutions for x , assuming a, b, c are constants with $a \neq 0$) with an equation in two variables (x, y) (which usually has infinitely many solutions). The equation given has in fact infinitely many solutions including $(\pm 1, 0)$, $(\pm 1766319049, \pm 226153980)$, etc.
5. No, there do not exist such integers m and n . The last two decimal digits of m^2 must be one of the 22 values

00, 01, 04, 09, 16, 21, 24, 25, 29, 36, 41, 44, 49, 56, 61, 64, 69, 76, 81, 84, 89, 96;

and the same for n^2 . There is no way to add two of these to get a number ending in 43. This argument is suggested by similar proofs in the textbook. A much simpler approach is to observe that every integer square has the form $4k$ or $4k+1$ for some integer k ; so a sum of two squares must have the form $4k$, $4k+1$, or $4k+2$. However, any number ending in '43' has the form $4k+3$.

6. Yes; the smallest example is $(m, n) = (19, 6)$. (There are infinitely many other solutions including $(721, 228)$, $(27379, 8658)$, ...)