

Discrete Structures

HW2

(Due 5:00pm, Thursday, October 1, 2026, on WyoCourses)

Instructions: See the syllabus for expectations regarding homework submissions. In particular, you are reminded that while discussion with other students is encouraged, your submissions must be your own work, not copied from any other sources, human or otherwise. You are also reminded that organization, presentation and clarity of solutions are all considered; proper notation, spelling, grammar, vocabulary and punctuation are required.

1. (20 points) Determine whether each of the following statements is true or false. Assume that the variables x, y, z are real-valued.
 - (a) For all $x > 0$, there exists $y > x$ such that $y < x^2$.
 - (b) There exists $x > 0$ such that every $y > x$ satisfies $y > x^2$.
 - (c) For all $x > 0$, there exists $y > x$ such that for some $z > x$, we have $z < y$.
 - (d) There exists $x > 0$ such that for every $y > x$, there exists $z < y$ such that $z > x$.
 - (e) There exists $x > 0$ such that for every $y > x$, there exists $z > y$ such that $z < x$.
 - (f) For some $x > 0$ and all $y > x$, there exists $z < y$ such that $z < x$.
 - (g) For all $x > 0$, if there exists $y > x$ such that $y < 1$, then $x < 1$.
 - (h) For all $x > 0$, there is some $y < x$ such that for all $z > y$, we have $z < x$.
 - (i) For all $x > 0$, there is some $y < x$ such that for all $z > y$, we have $z < x$.
 - (j) There is some $x > 0$ such that for all $y < x$ and all $z > y$, we have $z > x$.
2. (12 points) Evaluate each of the following sets in simplified form. Here we use the usual notation for intervals in $\mathbb{R}$, for example $(5, 8] = \{a \in \mathbb{R} \mid 5 < a \leq 8\}$ and $(-3, \infty) = \{a \in \mathbb{R} \mid a > -3\}$. Complements are in the ‘universe’ of real numbers; for example $\overline{(5, 8]} = (-\infty, 5] \cup (8, \infty)$. We denote integers by $m, n, \dots$ and real numbers by $a, b, x, y, \dots$
 - (a) $\bigcup_{n=1}^{\infty} (\frac{1}{n}, \infty)$
 - (b) $\bigcup_{n=1}^{\infty} [\frac{1}{n}, \infty)$
 - (c) $\overline{\bigcap_{n=1}^{\infty} (-\infty, \frac{1}{n})}$
 - (d) $\bigcup_{n \in \mathbb{Z}} [n, n+1]$
 - (e) $\bigcup_{m=0}^{\infty} \bigcap_{n=m+1}^{\infty} [m, n]$
 - (f) $\bigcap_{m=0}^{\infty} \bigcup_{n=m+1}^{\infty} [m, n]$

3. (12 points) Here $P(x)$ and $Q(y)$ are propositional functions. Match each of the propositions (I)–(VI) in the first column with its equivalent form (a)–(f) in the second column.

(I) $\neg(\forall x)(\forall y)(P(x) \rightarrow Q(y))$	(a) $(\exists x)(\forall y)((\neg P(x)) \wedge Q(y))$
(II) $(\forall x)(\exists y)(P(x) \wedge Q(y))$	(b) $(\exists x)(\exists y)(P(x) \wedge (\neg Q(y)))$
(III) $\neg(\forall x)(\exists y)(P(x) \vee (\neg Q(y)))$	(c) $(\forall y)(\forall x)(P(x) \vee Q(y))$
(IV) $(\forall x)(\forall y)((\neg P(x)) \rightarrow Q(y))$	(d) $(\exists y)(\exists x)((\neg Q(y)) \rightarrow P(x))$
(V) $(\exists x)(\exists y)(P(x) \vee Q(y))$	(e) $(\forall x)(P(x)) \wedge (\forall y)(Q(y))$
(VI) $(\forall x)(\forall y)(P(x) \wedge Q(y))$	(f) $(\forall x)(P(x)) \wedge (\exists y)(Q(y))$

4. (16 points) Which of the following sets are countable?

- (a) The set of all finite subsets of $\mathbb{Z}$.
- (b) The set of all subsets of $\mathbb{Z}$.
- (c) The set of all finite binary sequences (bitstrings of finite length, or words of finite length over the binary alphabet). This is the set $\{', 0, 1, 00, 01, 10, 11, 000, 001, 010, 011, \dots\}$.
- (d) The set of all binary sequences (i.e. sequences of 0's and 1's) which eventually repeat, e.g. sequences like $00010\overline{101} = 00010101101101101101 \dots$ where $\overline{101}$ indicates the repeating portion.
- (e) The set of all functions from $\mathbb{Z}$ to $\mathbb{Z}$.
- (f) The set of all functions from $\{0, 1\}$ to $\mathbb{R}$.
- (g) The set of all functions from $\{0, 1\}$ to $\mathbb{Z}$.
- (h) The set of all polynomials in x with rational coefficients, i.e. expressions such as $\frac{13}{5}x^5 + \frac{781}{40}x^3 - 19x - \frac{2}{81}$, where the degree can be any non-negative integer, and the coefficients are in $\mathbb{Q}$.