

Discrete Structures

Solutions to HW1

1. We identify the equivalent pairs of propositions as those which have the same truth table:

(I) and (d):

P	Q	I, d
T	T	T
T	F	T
F	T	F
F	F	F

(II) and (f):

P	Q	II, f
T	T	T
T	F	T
F	T	T
F	F	T

(III) and (a):

P	Q	III, a
T	T	T
T	F	F
F	T	F
F	F	T

(IV) and (b):

P	Q	IV, b
T	T	F
T	F	F
F	T	F
F	F	F

(V) and (c):

P	Q	V, c
T	T	F
T	F	T
F	T	F
F	F	T

(VI) and (e):

P	Q	VI, e
T	T	F
T	F	T
F	T	T
F	F	T

2. From the truth tables, we see that (II) and (f) are tautologies.
3. From the truth tables, we see that (IV) and (b) are contradictions.
4. The *exclusive 'or'* of P and Q , which is often written as $P \oplus Q$, can be written in terms of *our* logical operators in many different ways, such as $P \leftrightarrow (\neg Q)$ or $(\neg P) \leftrightarrow Q$.
5. (a) $V_0 = \emptyset$; $V_1 = \{\emptyset\}$; $V_2 = \{\emptyset, \{\emptyset\}\}$, $V_3 = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$.
 (b) V_n has $|V_n| = n$ elements; we write $|V_n| = n$.
 (c) The eight subsets of V_3 are $\emptyset$, $\{\emptyset\}$, $\{\{\emptyset\}\}$, $\{\{\emptyset, \{\emptyset\}\}\}$, $\{\emptyset, \{\emptyset\}\}$, $\{\emptyset, \{\emptyset, \{\emptyset\}\}\}$, $\{\{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$, $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$.
 In other words, the power set of V_3 is $\mathcal{P}(V_3) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\{\emptyset, \{\emptyset\}\}\}, \{\emptyset, \{\emptyset\}\}, \{\emptyset, \{\emptyset, \{\emptyset\}\}\}, \{\{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}, \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}\}$.
 (d) Since V_n has n elements, it has 2^n subsets. We write $|\mathcal{P}(V_n)| = 2^n$.