

Discrete Structures

HW1

(Due 5:00pm, Tuesday, September 15, 2026, on WyoCourses)

Instructions: See the syllabus for expectations regarding homework submissions. In particular, you are reminded that while discussion with other students is encouraged, your submissions must be your own work, not copied from any other sources, human or otherwise. You are also reminded that organization, presentation and clarity of solutions are all considered; proper notation, spelling, grammar, vocabulary and punctuation are required.

1. (24 points) Here P and Q are atomic propositions. Match each of the propositions (I)–(VI) in the first column with its equivalent form (a)–(f) in the second column.

(I) $(P \rightarrow Q) \rightarrow P$	(a) $P \leftrightarrow Q$
(II) $P \rightarrow (Q \rightarrow P)$	(b) $P \leftrightarrow (\neg P)$
(III) $(P \vee Q) \rightarrow (P \wedge Q)$	(c) $\neg Q$
(IV) $(P \wedge (\neg Q)) \leftrightarrow (P \rightarrow Q)$	(d) P
(V) $(P \rightarrow Q) \rightarrow (\neg Q)$	(e) $\neg(P \wedge Q)$
(VI) $(P \rightarrow Q) \rightarrow (\neg P)$	(f) $Q \rightarrow Q$

2. (5 points) In #1, which of the twelve propositions (I)–(VI) and (a)–(f) are tautologies?
3. (5 points) In #1, which of the twelve propositions (I)–(VI) and (a)–(f) are contradictions?
4. (6 points) The *exclusive ‘or’* of P and Q is true whenever exactly one of P and Q is true (not both). Note that this is distinct from the *inclusive ‘or’* $P \vee Q$. Express the exclusive ‘or’ of P and Q in terms of P and Q using the standard logical operators $\vee$, $\wedge$, $\neg$, $\rightarrow$, $\leftrightarrow$. Try to use the simplest expression you can (and so not all of these symbols will be required).
5. (20 points) Starting with the empty set $\emptyset = \{ \}$, we recursively construct a sequence of sets

$$V_0 = \emptyset;$$

$$V_{n+1} = V_n \cup \{V_n\}, \quad \text{for } n = 0, 1, 2, 3, \dots$$

- (a) Explicitly list V_n for $n = 0, 1, 2, 3$.
- (b) How many elements does V_n have?
- (c) Explicitly list all subsets of V_3 .
- (d) How many subsets does V_n have?