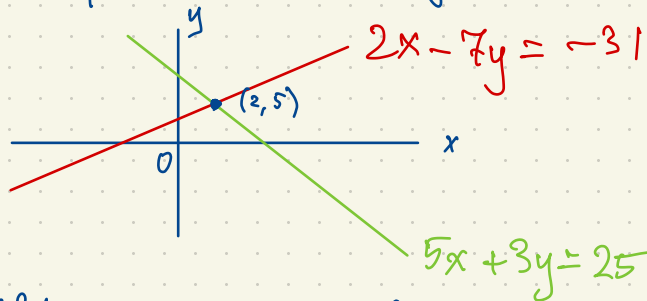


The background of the entire image is a repeating pattern of triangles. The triangles are arranged in a grid, with each row of triangles pointing downwards and each column of triangles pointing to the right. The triangles are a light beige or tan color, set against a white background.

Linear Algebra

Book 1

Example: Find all (x, y) such that $\underline{5x+3y=25}$ and $\underline{2x-7y=-31}$.



We are asking for the simultaneous solution of a system of two equations in two unknowns x and y .

$$\begin{cases} 5x + 3y = 25 & (1) \\ 2x - 7y = -31 & (2) \end{cases}$$

$$\begin{aligned} 41y &= 205 \\ y &= 5 \end{aligned}$$

$$\begin{aligned} 5x + 15 &= 25 \\ 5x &= 10 \\ x &= 2 \end{aligned}$$

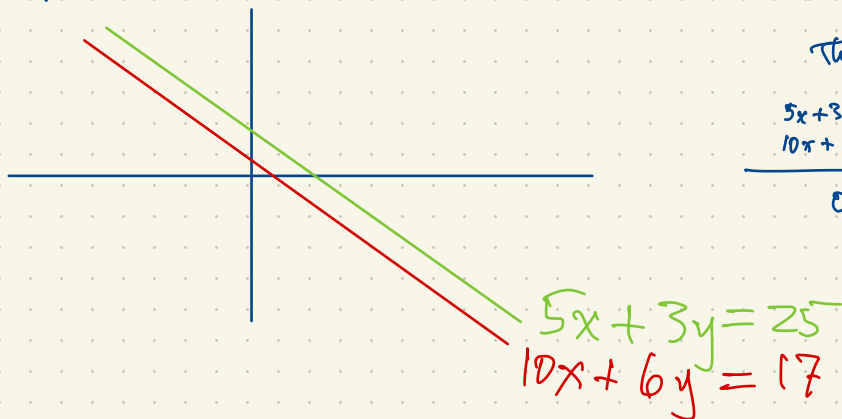
$$\begin{aligned} 2 \times (1) - 5 \times (2) &= (3) \\ (1) &= (3) \div 41 \end{aligned}$$

$$2 \times 3 - 5(-7) = 6 + 35 = 41$$

$$2 \times 25 - 5 \times (-31) = 50 + 155 = 205$$

Solution: $(x, y) = (2, 5)$ is the unique solution.

Example: Find all (x, y) such that $\underline{5x+3y=25}$ and $\underline{10x+6y=17}$.



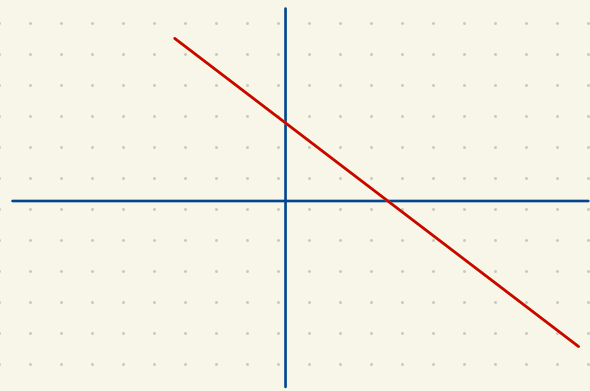
This system is inconsistent: it has no solution.

$$\begin{cases} 5x + 3y = 25 & (1) \\ 10x + 6y = 17 & (2) \end{cases}$$

$$0 = 33 \quad 2 \times (1) - (2)$$

This is inconsistent.

Example: Find all (x, y) such that $5x + 3y = 25$ and $15x + 9y = 75$.



$$5x + 3y = 25$$
$$15x + 9y = 75$$

This system is consistent but the solution is not unique: there are infinitely many solutions.

$$\begin{array}{r} 5x + 3y = 25 \quad (1) \\ 15x + 9y = 75 \quad (2) \\ \hline 0 = 0 \end{array} \quad (3) = 3 \times (1) - (2)$$