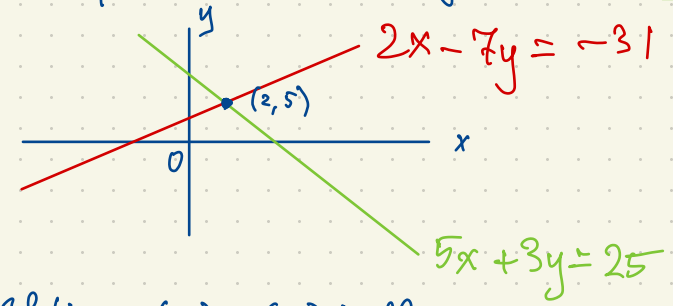
Linear Algebra

Book 1

Example: Find all (x,y) such that $5x+3y=25$ and $2x-7y=-31$.



We are asking for the simultaneous solution of a system of two equations in two unknowns x and y.

$$\begin{cases} 5x + 3y = 25 & (1) \\ 2x - 7y = -31 & (2) \end{cases}$$

$$41y = 205 \quad (4) = (3) \div 41$$

$$y = 5$$

$$2x(1) - 5x(2) = (3)$$

$$5x + 15 = 25$$

$$5x = 10$$

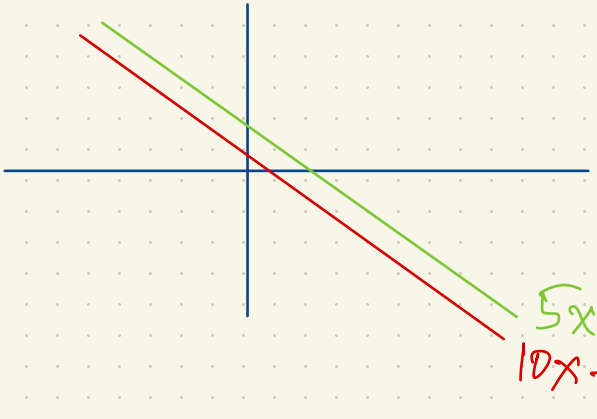
$$x = 2$$

$2\times3 - 5(-7) = 6 + 35 = 41$

$2\times25 - 5\times(-31) = 50 + 155 = 205$

Solution: $(x,y) = (2,5)$ is the unique solution.

Example: Find all (x,y) such that $5x+3y=25$ and $10x+6y=17$.



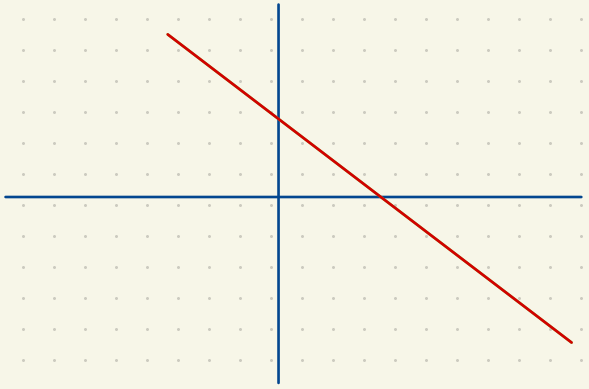
This system is inconsistent: it has no solution.

$$\begin{aligned} 5x+3y &= 25 \quad (1) \\ 10x+6y &= 17 \quad (2) \end{aligned}$$

$$0 = 33 \quad 2\times(1) - (2)$$

This is inconsistent.

Example: Find all (x,y) such that $5x+3y=25$ and $15x+9y=75$.



$5x+3y=25$

$15x+9y=75$

This system is consistent but the solution is not unique: there are infinitely many solutions.

$$\begin{aligned} 5x+3y &= 25 \quad (1)\\ 15x+9y &= 75 \quad (2)\\ \hline 0 &= 0 \quad (3) = 3\times(1)-(2) \end{aligned}$$