Name ....Solution Key........

UNIVERSITY OF WYOMING

Math 2250—Fall 2023

Department of Mathematics

Elementary Linear Algebra

Quiz 8

Friday, December 1, 2023

Given the matrix $A = \left[\begin{smallmatrix} 5 & -2 \\ 6 & -2 \end{smallmatrix}\right]$, find a basis $\mathbf{v}_1, \mathbf{v}_2$ of $\mathbb{R}^2$ consisting of eigenvectors of A. Also find the corresponding eigenvalues λ_1, λ_2 such that $A\mathbf{v}_i = \lambda_i \mathbf{v}_i$.

$$\lambda_1 = 1, \qquad \lambda_2 = 2, \qquad \mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

First determine the characteristic polynomial of A:

$$\det(A - \lambda I) = \begin{vmatrix} 5-\lambda & -2 \\ 6 & -2-\lambda \end{vmatrix} = (5-\lambda)(-2-\lambda) + 12 = \lambda^2 - 3\lambda + 2$$
$$= (\lambda - 1)(\lambda - 2)$$

whose roots 1, 2 give our eigenvalues.

v_1 is a null vector of $A - \lambda_1 I = A - I = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$; take $v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

v_2 " " " " " $A - \lambda_2 I = A - 2I = \begin{bmatrix} 3 & -2 \\ 6 & -4 \end{bmatrix}$; take $v_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

Check: $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ has $\operatorname{tr} D = 3$, $\det D = 2$;

$\operatorname{tr} A = 3$, $\det A = 2$.

$$\begin{bmatrix} 5 & -2 \\ 6 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}; \qquad \begin{bmatrix} 5 & -2 \\ 6 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}.$$