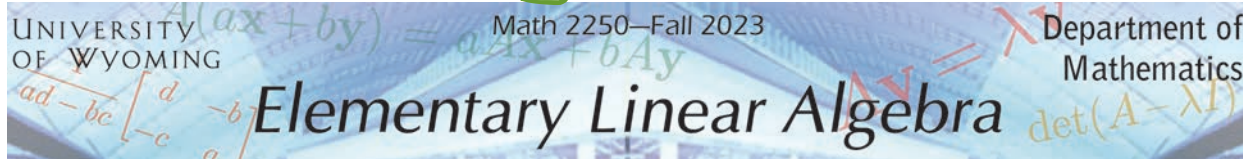


Name *Solution Key*



Quiz 8

Friday, December 1, 2023

Given the matrix $A = \begin{bmatrix} 5 & -2 \\ 6 & -2 \end{bmatrix}$, find a basis $\mathbf{v}_1, \mathbf{v}_2$ of \mathbb{R}^2 consisting of eigenvectors of A . Also find the corresponding eigenvalues λ_1, λ_2 such that $A\mathbf{v}_i = \lambda_i\mathbf{v}_i$.

$$\lambda_1 = 1, \quad \lambda_2 = 2, \quad \mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

First determine the characteristic polynomial of A :

$$\det(A - \lambda I) = \begin{vmatrix} 5-\lambda & -2 \\ 6 & -2-\lambda \end{vmatrix} = (5-\lambda)(-2-\lambda) + 12 = \lambda^2 - 3\lambda + 2 = (\lambda-1)(\lambda-2)$$

whose roots 1, 2 give our eigenvalues.

\mathbf{v}_1 is a null vector of $A - \lambda_1 I = A - I = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$; take $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

\mathbf{v}_2 " " " " " $A - \lambda_2 I = A - 2I = \begin{bmatrix} 3 & -2 \\ 6 & -4 \end{bmatrix}$; take $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

Check: $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ has $\text{tr } D = 3$, $\det D = 2$;
 $\text{tr } A = 3$, $\det A = 2$.

$$\begin{bmatrix} 5 & -2 \\ 6 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}; \quad \begin{bmatrix} 5 & -2 \\ 6 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}.$$