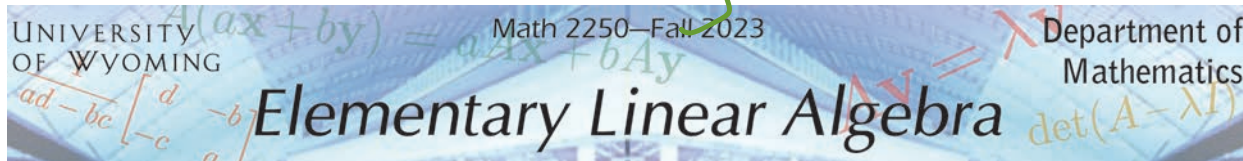


Name *Solution Key*

UNIVERSITY OF WYOMING

Math 2250—Fall 2023

Department of Mathematics



Elementary Linear Algebra

Quiz 6

Friday, October 13, 2023

FRIDAY THE 13TH

Consider the following 3×3 matrix and column vectors in \mathbb{R}^3 :

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}.$$

Compute the inverse matrix A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array} \right] \sim \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array} \right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 2 & -1 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array} \right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -1 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array} \right] \quad \text{so } A^{-1} = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}.$$

Use your inverse matrix, as found above, to solve the linear system $A\mathbf{x} = \mathbf{b}$.

$$\text{If } A\mathbf{x} = \mathbf{b} \text{ then } \mathbf{x} = A^{-1}\mathbf{b} = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \\ 4 \end{bmatrix}.$$

Check that your solution for \mathbf{x} does solve the linear system.

$$A\mathbf{x} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} -3 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}.$$

Yes, \mathbf{x} satisfies the system $A\mathbf{x} = \mathbf{b}$.