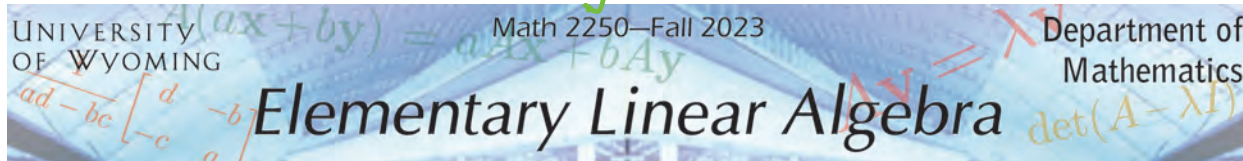Name .... *Solution Key* ....

Quiz 5

Friday, October 6, 2023

Consider the following 3×3 matrix and four column vectors in $\mathbb{R}^3$:

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 2 & 5 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{u}_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{u}_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

Indicate whether each of the following statements is True or False:

1. The span of the vectors $\mathbf{u}_1, \mathbf{u}_2$ contains the vector $\mathbf{u}_3$.   _____(True/**False**)

2. The span of the vectors $\mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4$ contains the vector $\mathbf{u}_1$.   _____(**True**/False)

    $u_1 = \frac{1}{5} u_2 + 0 u_3 + 0 u_4$

3. The span of the vectors $\mathbf{u}_1, \mathbf{u}_2$ consists of a plane through the origin in $\mathbb{R}^3$.   _____(True/**False**)

    Span $\{u_1, u_2\}$ is only a line through the origin in $\mathbb{R}^3$ (the x-axis).

4. The span of the vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$ consists of a plane through the origin in $\mathbb{R}^3$.   _____(**True**/False)

5. The span of the vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4$ consists of a plane through the origin in $\mathbb{R}^3$.   _____(True/**False**)

    Span $\{u_1, u_2, u_3, u_4\}$ is all of $\mathbb{R}^3$.

6. The span of the columns of A contains the vector $\mathbf{u}_3$.   _____(**True**/False)

    Take the first column of A, plus half of the second column.

7. The span of the columns of A contains the vector $\mathbf{u}_4$.   _____(True/**False**)

    u_4 lies outside of the xy-plane.

8. The span of the columns of A consists of a plane through the origin in $\mathbb{R}^3$.   _____(**True**/False)

9. Every plane through the origin in $\mathbb{R}^3$ can be spanned by a set of two vectors.   _____(**True**/False)

10. For every vector $\mathbf{v}$ in $\mathbb{R}^3$, the vector $A\mathbf{v}$ is a linear combination of the columns of A.   _____(**True**/False)

    Actually, this is true for every matrix with three rows.