Name ......... Solution Key .........

UNIVERSITY OF WYOMING — Math 2250—Fall 2023 — Department of Mathematics

# Elementary Linear Algebra

## Quiz 4

Friday, September 29, 2023

Consider the following five vectors in $\mathbb{R}^3$:

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{u}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{u}_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{u}_5 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

Indicate whether each of the following statements is True or False:

1. The vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$ are linearly dependent. _____*(True/False)*

   $u_1 = 2u_2 - u_3$

2. The vectors $\mathbf{u}_1, \mathbf{u}_3, \mathbf{u}_5$ are linearly dependent. _____*(True/False)*

   $u_1 \neq 0$, $u_3$ is not a scalar multiple of $u_1$, and $u_5$ is not a linear combination of $u_1$ and $u_3$.

3. The vector $\mathbf{u}_5$ is a linear combination of the vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4$. _____*(True/False)*

   Look at the third coordinate!

4. The vector $\mathbf{u}_4$ is a linear combination of the vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_5$. _____*(True/False)*

   $u_4 = -u_1 + u_2 + 0u_3 + 0u_5$

5. The vectors $\mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_5$ are linearly independent. _____*(True/False)*

6. The vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_4$ are linearly independent. _____*(True/False)*

   $-u_1 + u_2 - u_4 = 0$

7. The vector $\mathbf{u}_1$ is a linear combination of the vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_5$. _____*(True/False)*

   $u_1 = 1u_1 + 0u_2 + 0u_5$

8. If $S$ and $S'$ are sets of vectors in $\mathbb{R}^n$, and every vector in $S$ is a linear combination of vectors in $S'$, then every vector in $S'$ must also be a linear combination of vectors in $S$. e.g. in $\mathbb{R}^3$: $S = \{u_1, u_2\}$, $S' = \{u_1, u_2, u_5\}$ _____*(True/False)*

9. If $S \subseteq S'$ for some sets of vectors $S, S' \subseteq \mathbb{R}^n$, and $S$ is linearly independent, then $S'$ must also be linearly independent. _____*(True/False)*

   eg. in $\mathbb{R}^3$, $S = \{u_1, u_2\}$, $S' = \{u_1, u_2, u_3\}$

10. If $S \subseteq S'$ for some sets of vectors $S, S' \subseteq \mathbb{R}^n$, and every vector $\mathbf{v} \in \mathbb{R}^n$ is a linear combination of vectors in $S$, then every $\mathbf{v} \in \mathbb{R}^n$ must also be a linear combination of vectors in $S'$. _____*(True/False)*

    Let $S = \{v_1, v_2, \dots, v_k\}$, $S' = \{v_1, v_2, \dots, v_k, v_{k+1}, \dots, v_m\}$.

    If $v \in \mathbb{R}^n$ has the form $v = c_1v_1 + \dots + c_kv_k$, then $v = c_1v_1 + \dots + c_kv_k + 0v_{k+1} + \dots + 0v_m$.