University of Wyoming
Department of Mathematics

Math 2250—Fall 2023

Elementary Linear Algebra

HW3

(Due Monday, December 4, 2023 by 5:00pm on WyoCourses)

Instructions: Work by hand. Show your work. Always check your answers wherever feasible (you may use software to *check* your work). Write clearly, using complete sentences where appropriate, and always using correct vocabulary, spelling, punctuation and notation. Total value of questions: 85 points.

1. (*30 points*) Compute each of the following determinants, in simplified form:

(a) $\begin{vmatrix} a & 2a+b \\ b & a+2b \end{vmatrix}$ (b) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 3 & 4 \\ 1 & 9 & 16 \end{vmatrix}$ (c) $\begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{vmatrix}$

2. (*30 points*) Consider the matrix

$$A = \begin{bmatrix} -7 & -5 & 1 \\ 8 & 7 & -2 \\ 8 & 5 & 0 \end{bmatrix}.$$

(a) Determine the characteristic polynomial of A.

(b) Find the eigenvalues $\lambda_1, \lambda_2, \lambda_2$ of A.

(c) Find a basis $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ consisting of eigenvectors $\mathbb{R}^3$ corresponding to the eigenvalues found in (b), i.e. $A\mathbf{v}_i = \lambda_i \mathbf{v}_i$ for $i = 1, 2, 3$.

3. (*25 points*) Consider the vector space of all polynomials in x of degree at most 4, with real coefficients:

$$V = \{a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 \,:\, a_0, a_1, a_2, a_3, a_4 \in \mathbb{R}\}.$$

Note that V has dimension 5; and the standard basis for V is $\{1, x, x^2, x^3, x^4\}$.

(a) Let U be the subspace consisting of all $f(x) \in V$ such that $f(1) = f(3) = 0$. Find a basis for U.

(b) Determine the dimension of U.

(c) Consider the linear transformation $T : V \to V$ defined by $T(f(x)) = f(x+1)$. Find the matrix expressing T (with respect to the standard basis listed above).