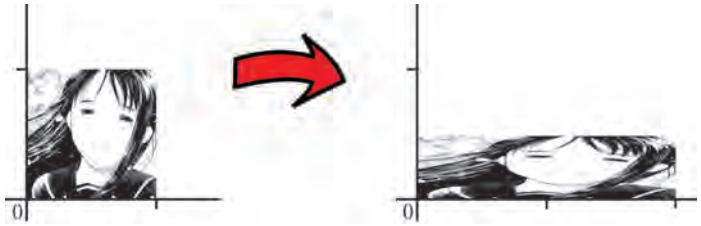
HW2

(Due Monday, October 30, 2023 by 5:00pm on WyoCourses)

Instructions: Work by hand. Show your work. Always check your answers wherever feasible. Write clearly, using complete sentences where appropriate, and always using correct notation. For further instructions, see the syllabus and the FAQ's linked there. Total value of questions: 85 points.

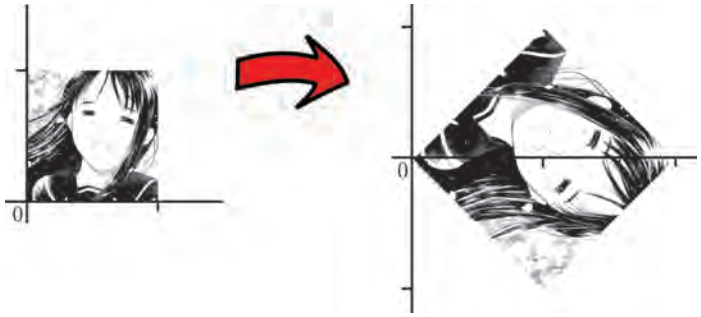
1. (*20 points*) In each of the four cases below, a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is depicted by its effect on a square reference image. Write down the standard 2×2 matrix of the linear transformation in each case. Note that in (c), T reverses orientation. Refer to examples done in class on Oct 2.

(a)

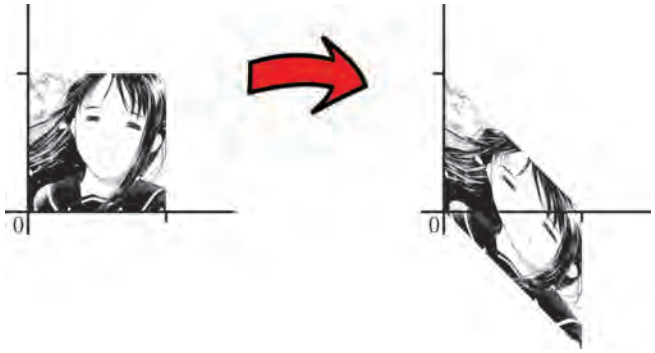


(b)

(c)



(d)



2. (*30 points*) Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 4 & 4 & -2 & -1 \\ 0 & 0 & 1 & 3 & -1 & -3 \\ -1 & -2 & -1 & 1 & 3 & 0 \\ 2 & 4 & 7 & 4 & -2 & 3 \end{bmatrix}.$$

(a) Find a basis for $\operatorname{Col} A$, the column space of A (the span of the columns of A).
(b) What is the dimension of $\operatorname{Col} A$?
(c) Find a basis for $\operatorname{Nul} A$, the null space of A (the set of solutions of $A\mathbf{x} = \mathbf{0}$).
(d) What is the dimension of $\operatorname{Nul} A$?
(e) Find a basis for $\operatorname{Row} A$, the row space of A (the span of the rows of A).
(f) What is the dimension of $\operatorname{Row} A$?

3. (*15 points*) The column vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 3 \end{bmatrix}$ span a subspace $U \subset \mathbb{R}^4$. Find a 2×4 matrix A having U as its null space.

4. (*20 points*) Let V be the set of all polynomials in x of degree at most 3, i.e. V is the set of all polynomials of the form $p(x) = ax^3 + bx^2 + cx + d$ where $a, b, c, d \in \mathbb{R}$. The linear transformation $T : V \to \mathbb{R}^4$, which evaluates each polynomial at the four points $0, 1, 2, 3$, is represented by a 4×4 matrix A. Refer to the example done in class on Oct 13 for interpolating three data points using a polynomial of degree 2.

(a) Working by hand, compute the inverse matrix A^{-1} by the method demonstrated in class.
(b) Using (a), find the unique polynomial $p(x) \in V$ of degree 3 whose graph passes through the four points $(0, 3)$, $(1, -3)$, $(2, -5)$, $(3, 15)$.