



HW1

(Due Monday, September 25, 2023 by 5:00pm on WyoCourses)

Instructions: Work by hand. Show your work. Always check your answers wherever feasible. Write clearly, using complete sentences where appropriate, and always using correct notation. For further instructions, see the syllabus and the FAQ's linked there. Total value of questions: 50 points.

1. (10 points) A linear system of three equations in six unknowns x_1, x_2, \dots, x_6 is represented by the matrix

$$\left[\begin{array}{cccccc|c} 0 & 1 & 0 & 2 & -1 & 0 & -4 \\ 0 & 0 & 1 & 1 & 5 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 6 \end{array} \right].$$

- (a) Which of the unknowns are *basic* variables? and which are *free* variables? (see p.18).
(b) Write down the general solution of the linear system in terms of parameters r, s, t .
2. (10 points) You are given the matrix

$$A = \begin{bmatrix} 2 & 4 & -1 & -6 & 1 & 14 \\ 1 & 2 & 3 & -3 & 4 & 7 \\ 1 & 2 & -1 & -3 & 0 & 7 \end{bmatrix}.$$

Using a sequence of elementary row operations, reduce your answer to reduced row echelon form.

3. (10 points) Consider the matrix $A = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$. Evaluate A^2 , A^3 and A^{100} .
4. (10 points) Consider the matrix $B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. Evaluate B^2 , B^3 , B^4 , B^5 and B^{99} .
5. (10 points) Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$. Find the set of all matrices that commute with A . (To do this, you must find all matrices X satisfying $AX = XA$ by solving the appropriate linear system.)