Math 2200-01 (Calculus I) Spring 2020

Book 1

Jan 27

Calculus I : Single-variable calculus $y = f(x)$ for example (one input variable x, one output variable). Derivatives (rates of change) : <u>differential</u> calculus.

Calculus II : also single-variable. <u>Integral</u> calculus.

Calculus III : multivariable ie. several input variables and/or several output variables

<u>eg</u>. position $(x(t), y(t), z(t))$ of an object at time t: one input t, three output variables $x(t), y(t), z(t)$.

<u>Eg</u>. Temperature in this room as a function of position $T(x,y,z)$ (three inputs x,y,z; one output T)

<u>Eg</u>. Wind velocity as a function of position : three inputs x,y,z; three outputs are the components of wind velocity.

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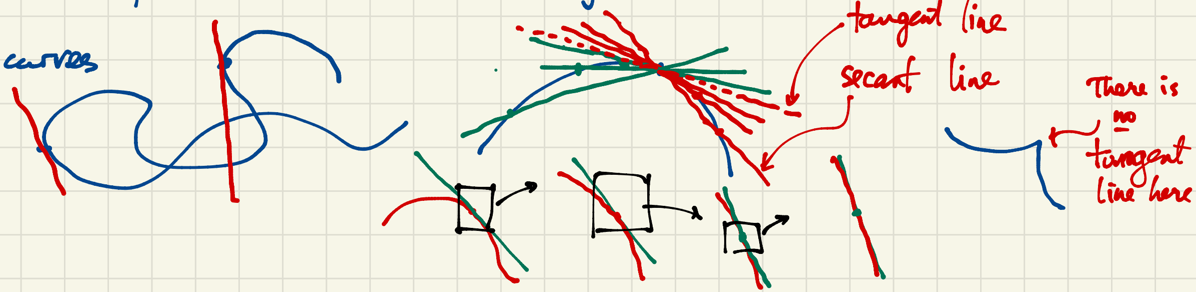
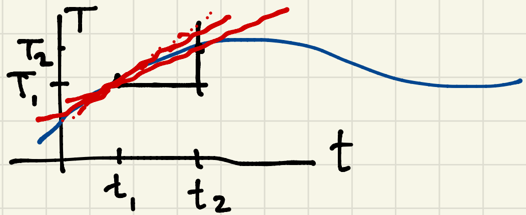
Tangent lines to curves





Temperature T as a function of time t

During the time interval $[t_1, t_2]$ i.e. $t_1 \le t \le t_2$ the temperature rises from T_1 to T_2.

The <u>average</u> rate of change of temperature during this time interval is

$$\frac{\Delta T}{\Delta t} = \frac{T_2 - T_1}{t_2 - t_1}$$

$T_2 - T_1 \leftarrow$ change in temperature

$t_2 - t_1 \leftarrow$ time elapsed.

= slope of the secant line from (t_1, T_1) to (t_2, T_2) on the graph.

We want to understand the <u>instantaneous</u> rate of change of temperature at time t_1. To determine this, first consider the <u>average</u> rate of change over smaller and smaller time intervals $[t_1, t_2]$ where we take $t_2 \to t_1$ (t_2 gets closer and closer to t_1).

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Eg.

t_2	$\frac{T_2 - T_1}{t_2 - t_1}$
4	2 degrees/hour
3.2	2.17
3.1	2.19
3.001	2.197
2.9	2.204
2.7	2.23
2	2.31

In my example, $t_1 = 3$.

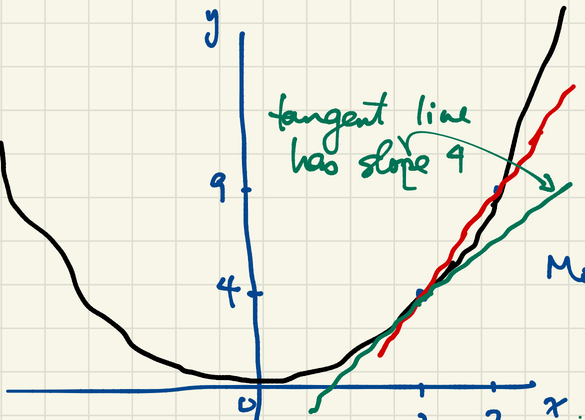
The limit is 2.2.

(The temperature at 3pm is changing at a rate of 2.2 degrees per hour.

We write $\lim_{t_2 \to 3} \frac{T_2 - T_1}{t_2 - t_1} = 2.2$

(the limit of $\frac{T_2 - T_1}{t_2 - t_1}$ is 2.2 as t_2 approaches 3).

A second example using a polynomial function $y = f(x) = x^2$. Find the rate of change of y with respect to x at $x = 2$.



The secant line joining the points $(2,4)$ and $(3,9)$ on the curve has slope

$$\frac{\Delta y}{\Delta x} = \frac{9-4}{3-2} = 5.$$

More generally, if we join the points $(2,4)$ and $(x, f(x))$ on the curve, the secant line has slope

$f(2) = 4$

$$\frac{\Delta y}{\Delta x} = \frac{f(x) - f(2)}{x - 2}$$

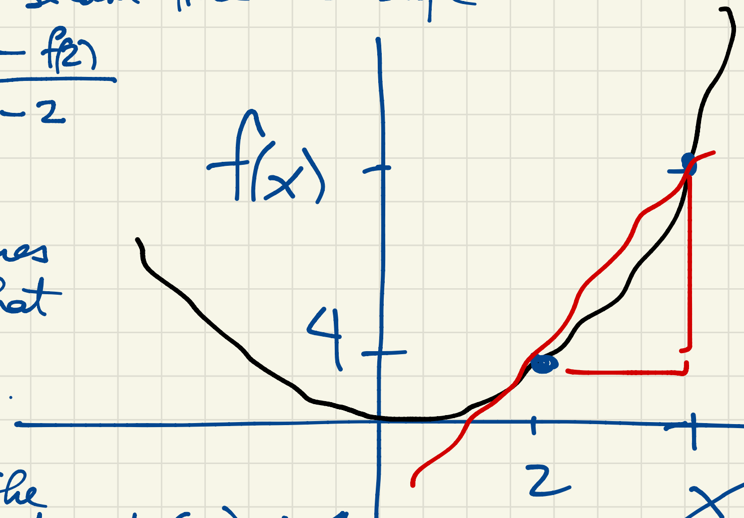
The tangent line at $(2,4)$ is $y - 4 = 4(x-2)$ i.e. $y = 4x - 4$.

x	$\dfrac{f(x)-f(2)}{x-2}$
3	5
2.5	4.5
2.1	4.1
2.01	4.01
1.99	3.99
1.9	3.9
1.5	3.5
1	3

Based on the table of values we guess that

$$\lim_{x \to 2} \frac{f(x) - f(2)}{x-2} = 4.$$

i.e. the slope of the tangent line to the graph at $(2,4)$ is 4.



If a function has a sufficiently nice formula eg. polynomial, then we have algebraic rules that provide definite ways to evaluate limits, eliminating guesswork based on the graph or table of values.

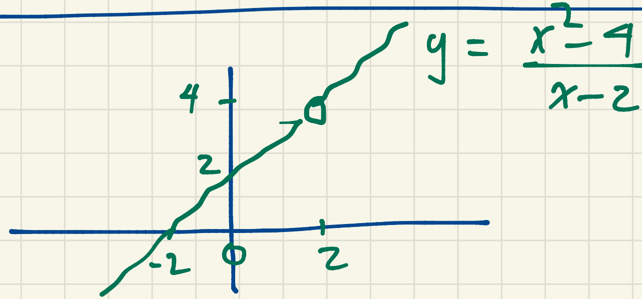
Eg. Find the slope of the tangent line to the graph of $y=x^2$ at $(2,4)$.

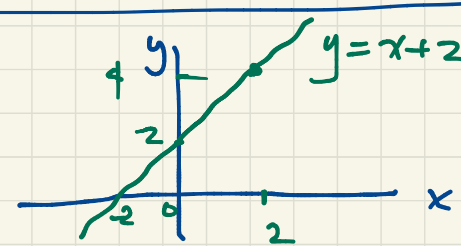
Solution The secant line from $(2,4)$ to $(x, f(x)) = (x, x^2)$ has slope

$$\frac{\Delta y}{\Delta x} = \frac{x^2-4}{x-2} = \frac{(x+2)(x-2)}{x-2} = x+2. \quad \text{for } x \neq 2.$$

The slope of the tangent line is

$$\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} (x+2) = 2+2 = 4.$$





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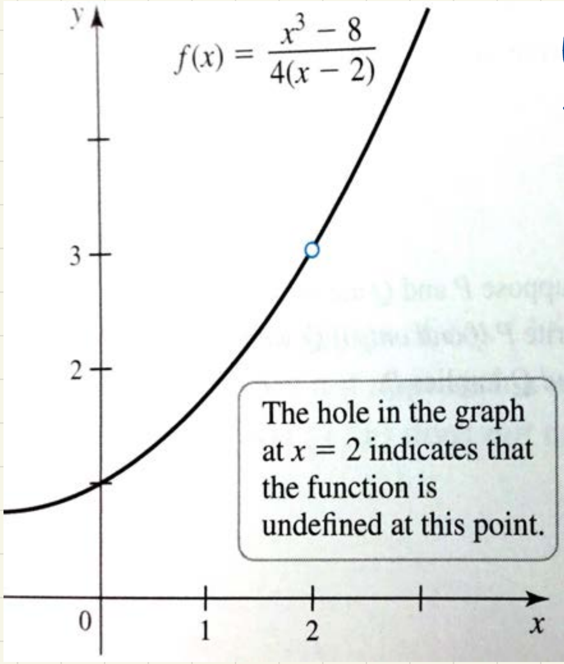
Both of these functions satisfy $\lim_{x\to 2} f(x) = 4$



$$\lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{x^3 - 8}{4(x-2)} = 3$$