Math 2200-01 (Calculus I) Spring 2020

Book 1

Jan 27

Calculus I : Single-variable calculus $y = f(x)$ for example (one input variable x, one output variable). Derivatives (rates of change) : <u>differential</u> calculus.

Calculus II : also single-variable. <u>Integral</u> calculus.

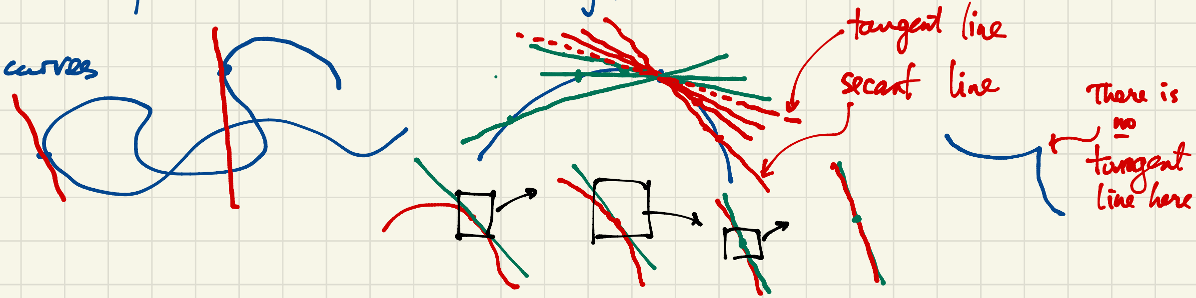
Calculus III : multivariable ie. several input variables and/or several output variables
<u>eg</u>. position $(x(t), y(t), z(t))$ of an object at time t: one input t, three output variables $x(t), y(t), z(t)$.

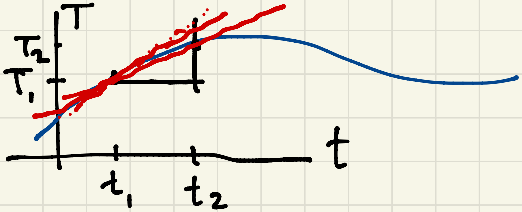
<u>Eg</u>. Temperature in this room as a function of position $T(x,y,z)$
(three inputs x,y,z; one output T)

<u>Eg</u>. Wind velocity as a function of position: three inputs x,y,z; three outputs are the components of wind velocity.

Jan 28

Tangent lines to curves





Temperature T as a function of time t

During the time interval $[t_1, t_2]$ i.e. $t_1 \le t \le t_2$ the temperature rises from T_1 to T_2.

The average rate of change of temperature during this time interval is

$$\frac{\Delta T}{\Delta t} = \frac{T_2 - T_1}{t_2 - t_1}$$

$T_2 - T_1$ ← change in temperature

$t_2 - t_1$ ← time elapsed.

= slope of the secant line from (t_1, T_1) to (t_2, T_2) on the graph.

We want to understand the instantaneous rate of change of temperature at time t_1. To determine this, first consider the average rate of change over smaller and smaller time intervals $[t_1, t_2]$ where we take $t_2 \to t_1$ (t_2 gets closer and closer to t_1).

Eg.

In my example, $t_1 = 3$.

t_2	$\frac{T_2 - T_1}{t_2 - t_1}$	
4	2	degrees/hour
3.2	2.17	
3.1	2.19	
3.001	2.197	
2.9	2.204	
2.7	2.23	
2	2.31	

The limit is 2.2.