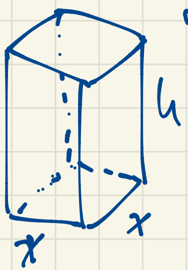
Math 2200-01 (Calculus I) Spring 2020

Book 3

Sec 4.5 : Optimization

April 13

p.285 #19. Of all boxes with a square base and a volume $8\,m^3$, which one has the minimum surface area?



volume $V = x^2 h = 8 \implies h = \frac{8}{x^2}$

area $A = \underbrace{2x^2}_{\text{top and bottom}} + \underbrace{4xh}_{\text{sides}} = 2x^2 + 4x \cdot \frac{8}{x^2} = 2x^2 + \frac{32}{x}$, $x > 0$

The domain is $(0, \infty)$, an unbounded open interval.

$$\frac{dA}{dx} = 4x - \frac{32}{x^2} = \frac{4}{x^2}\left(x^3 - 8\right)$$

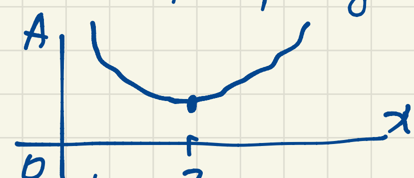
The critical point is at $x = 2$.

(Recall: critical points are where the derivative is zero or undefined. There are no points of the domain where $\frac{dA}{dx}$ is undefined, only one point where $\frac{dA}{dx} = 0$.)

For $0 < x < 2$, $\frac{dA}{dx} < 0$ so $A(x)$ is decreasing.

For $x = 2$, $\frac{dA}{dx} = 0$.

For $x > 2$, $\frac{dA}{dx} > 0$ so $A(x)$ is increasing.



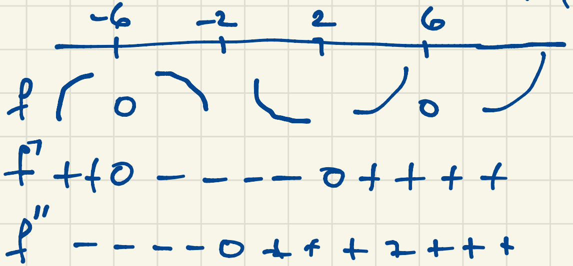
So the minimum surface area $A(2) = 12\,m^2$ occurs for a box of size $2m \times 2m \times 2m$.

Sec 4.4. p.278 # 21.

$f(x) = (x-6)(x+6)^2 = (x^2-36)(x+6) = x^3+6x^2-36x-216$

$f'(x) = 3x^2+12x-36 = 3(x^2+4x-12) = 3(x+6)(x-2)$

$f''(x) = 6x+12 = 6(x+2)$



$f(-2) = (-8)(4)^2 = -128$

$f(2) = (-4)(8^2) = -256$