Name Solution Key

UNIVERSITY OF WYOMING | Math 2200—Spring 2020 | Department of Mathematics

Calculus I

Quiz 9—Friday, April 24

Determine each of the following limits. You may use l'Hôpital's Rule if it is appropriate; or you may use previous methods. However you must show your work. Answers must be justified analytically, not based on numerical guesses.

1. $\lim_{x\to 0} \dfrac{\sin(3x)\cos(5x)}{x}$ $= \lim_{x\to 0} \dfrac{3\cos 3x\cos 5x - 5\sin 3x\sin 5x}{1} = \dfrac{3-0}{1}$

$= 3$ using l'Hôpital's Rule.

OR $\lim_{u\to 0} \dfrac{\sin u}{u/3} \cdot \lim_{x\to 0} \cos 5x = \dfrac{1}{1/3}\cdot 1 = 3$ the "old" way.

2. $\lim_{x\to\infty} \dfrac{(\ln x)^3}{x^2} = \lim_{x\to\infty} \dfrac{3(\ln x)^2\cdot\frac{1}{x}}{2x}$ by l'Hôpital's Rule

$= \dfrac{3}{2}\lim_{x\to\infty}\left(\dfrac{\ln x}{x}\right)^2 = \dfrac{3}{2}\cdot 0^2 = 0.$

Here I have made use of the previous limit we have demonstrated.

$$\lim_{x\to\infty}\frac{\ln x}{x} = \lim_{x\to\infty}\frac{1/x}{1} = 0.$$

A much faster solution uses the substitution $y = \ln x$:

$$\lim_{x\to\infty}\frac{(\ln x)^3}{x^2} = \lim_{y\to\infty}\frac{y^3}{e^{2y}} = \lim_{y\to\infty}\frac{3y^2}{2e^{2y}} = \lim_{y\to\infty}\frac{6y}{4e^{2y}}$$

$$= \lim_{y\to\infty}\frac{6}{8e^{2y}} = 0.$$

Here's how it might work if you blindly proceed with three applications of l'Hôpital's Rule to the original limit:

$$\lim_{x\to\infty} \frac{(\ln x)^3}{x^2} = \lim_{x\to\infty} \frac{3(\ln x)^2 \cdot \frac{1}{x}}{2x} \quad \text{by l'Hôpital's Rule}$$

$$= \lim_{x\to\infty} \frac{3(\ln x)^2}{2x^2} \quad \text{after simplifying}$$

$$= \lim_{x\to\infty} \frac{(6\ln x)\frac{1}{x}}{4x} \quad \text{by l'Hôpital's Rule}$$

$$= \lim_{x\to\infty} \frac{3\ln x}{2x^2} \quad \text{after simplifying}$$

$$= \lim_{x\to\infty} \frac{3\cdot\frac{1}{x}}{4x} \quad \text{by l'Hôpital's Rule}$$

$$= \lim_{x\to\infty} \frac{3}{4x^2} \quad \text{after simplifying}$$

$$= 0.$$