Name . . . . *Solution Key* . . . .
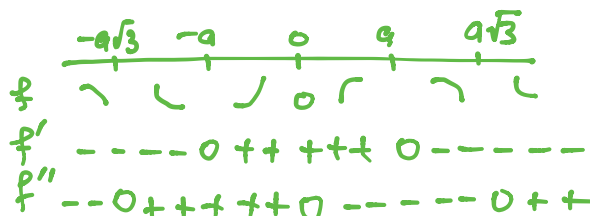
Optional Quiz 8—Monday, April 27

If you complete this quiz (today only), I will record the result and drop the lowest of your quiz scores. Consider the function $f(x) = \dfrac{x}{x^2+a^2}$ where a is a positive constant.

a) Determine both $f'(x)$ and $f''(x)$ in simplified form.

$$f'(x) = \frac{(x^2+a^2) - x\cdot 2x}{(x^2+a^2)^2} = \frac{a^2-x^2}{(x^2+a^2)^2} = \frac{(a+x)(a-x)}{(x^2+a^2)^2}$$
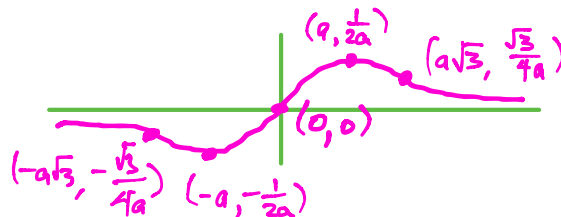
$$f''(x) = \frac{(x^2+a^2)^2(-2x) - 2(x^2+a^2)(2x)(a^2-x^2)}{(x^2+a^2)^4} = \frac{2x(x^2-3a^2)}{(x^2+a^2)^3}$$

$$= \frac{2x(x+a\sqrt{3})(x-a\sqrt{3})}{(x^2+a^2)^3}$$

b) List intervals where f is increasing.

$$(-a, a)$$

c) List intervals where f is decreasing.

$$(-\infty, -a), \quad (a, \infty)$$

d) List intervals where the graph of f is concave up.

$$(a\sqrt{3}, \infty), \quad (-a\sqrt{3}, 0)$$

(OVER)

e) List intervals where the graph of f is concave down.

$$(-\infty, a\sqrt{3}), \quad (0, a\sqrt{3})$$

f) Which kind of symmetry does the graph of f have?

Since $f(-x) = -f(x)$, f is an odd function.
(Its graph is symmetric about the origin under $180°$ rotation).

g) List the coordinates of the absolute maximum point of f.

$$\left(a, \tfrac{1}{2a}\right)$$

h) List the coordinates of the absolute minimum point of f.

$$\left(-a, -\tfrac{1}{2a}\right)$$

i) List all inflection points of the graph of f.

$$(0,0), \quad \left(a\sqrt{3}, \tfrac{\sqrt{3}}{4a}\right), \quad \left(-a\sqrt{3}, -\tfrac{\sqrt{3}}{4a}\right)$$

j) List all asymptotes of the graph of f.

The x-axis (a horizontal asymptote which is the line $y=0$).
This follows from $\lim\limits_{x \to \infty} f(x) = \lim\limits_{x \to -\infty} f(x) = 0$.