Sec 4.7

#32. Using l'Hôpital's Rule, $\lim_{z\to 0} \frac{\tan 4z}{\tan 7z} = \lim_{z\to 0} \frac{4\sec^2 4z}{7\sec^2 7z} = \frac{4\cdot 1}{7\cdot 1} = \frac{4}{7}$.

Alternatively, $\lim_{z\to 0} \frac{\tan 4z}{\tan 7z} = \lim_{z\to 0} \frac{4}{7}\cdot\frac{(\sin 4z)/4z}{(\sin 7z)/7z}\cdot\frac{\cos 7z}{\cos 4z} = \frac{4}{7}\cdot\frac{1}{1}\cdot\frac{1}{1} = \frac{4}{7}$.

#34. Without l'Hôpital's Rule, substitute $y=3x$:

$$\lim_{x\to 0}\frac{\sin^2 3x}{x^2} = \lim_{y\to 0}\frac{\sin^2 y}{(y/3)^2} = \lim_{y\to 0} 9\left(\frac{\sin y}{y}\right)^2 = 9\cdot 1 = 9.$$

With l'Hôpital's Rule,

$$\lim_{x\to 0}\frac{\sin^2 3x}{x^2} = \left(\lim_{x\to 0}\frac{\sin 3x}{x}\right)^2 = \left(\lim_{x\to 0}\frac{3\cos 3x}{1}\right)^2 = \left(\frac{3\cdot 1}{1}\right)^2 = 9.$$

#36. Using l'Hôpital's Rule,

$$\lim_{x\to 0}\frac{e^x - x - 1}{5x^2} = \lim_{x\to 0}\frac{e^x - 1}{10x} = \lim_{x\to 0}\frac{e^x}{10} = \frac{1}{10}.$$

#38. Without l'Hôpital's Rule,

$$\lim_{x\to\infty}\frac{e^{3x}}{3e^{3x}+5} = \lim_{x\to\infty}\frac{1}{3+\frac{5}{e^{3x}}} = \frac{1}{3+0} = \frac{1}{3}.$$

#40. Using l'Hôpital's Rule,

$$\lim_{x\to 0}\frac{\sin x - x}{7x^3} = \lim_{x\to 0}\frac{\cos x - 1}{21x^2} = \lim_{x\to 0}\frac{-\sin x}{42x} = \lim_{x\to 0}\frac{-\cos x}{42} = -\frac{1}{42}$$

#52. Without l'Hôpital's Rule, substitute $x = 1+t^2$ where $t\to 0^+$:

$$\lim_{x\to 1^+}\left(\frac{1}{x-1} - \frac{1}{\sqrt{x-1}}\right) = \lim_{t\to 0^+}\left(\frac{1}{t^2} - \frac{1}{t}\right) = \lim_{t\to 0^+}\frac{1-t}{t^2} = \infty.$$

#54. With l'Hôpital's Rule,

$$\lim_{x \to 1^-} (1-x) \tan \frac{\pi x}{2} = \lim_{x \to 1^-} \frac{(1-x) \sin \frac{\pi x}{2}}{\cos \frac{\pi x}{2}}$$

$$= \lim_{x \to 1^-} \frac{-\sin \frac{\pi x}{2} + \frac{\pi}{2}(1-x) \cos \frac{\pi x}{2}}{-\frac{\pi}{2} \sin \frac{\pi x}{2}} = \frac{-1+0}{-\frac{\pi}{2}} = \frac{2}{\pi}.$$