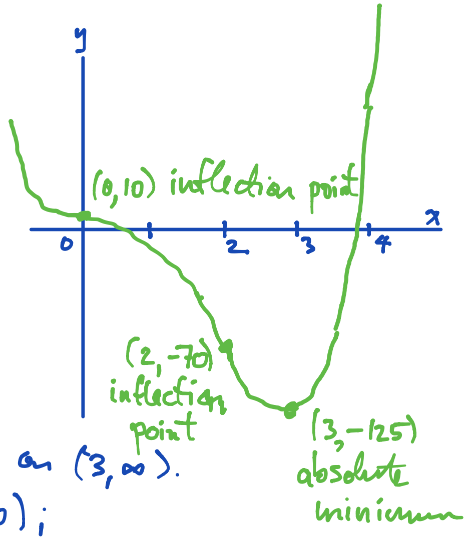
Sec 4.3

#65. $f(x) = 5x^4 - 20x^3 + 10$

$f'(x) = 20x^3 - 60x^2 = 20x^2(x-3)$

$f''(x) = 60x^2 - 120x = 60x(x-2)$

	0		2		3	
f'	$- -$	0	$- - - -$		$- -$ 0	$+ +$
f''	$+ +$	0	$- - -$	0	$+ + +$	$+ +$



f is decreasing on $(-\infty, 3)$ and increasing on $(3, \infty)$.

f is concave up on $(-\infty, 0)$ and on $(2, \infty)$; it is concave down on $(0, 2)$.

The graph has two inflection points as shown. The critical point $(0, 10)$ is not a local extremum; it is an inflection point where the tangent line happens to be horizontal.

The only other critical point, $(3, -125)$, is a local minimum; it is also the absolute minimum point.

Since $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = \infty$, there is no global maximum point.

There is no local maximum, no asymptotes and no evident symmetry (f is neither even nor odd).

The y-intercept is 10. The x-intercepts are at approximately 0.8604 and 3.968.

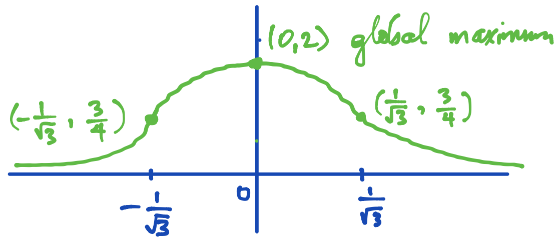
N.B. I forgot that in Sec 4.3 we weren't asked for this much detail about the function! So my answers look much more like the answers from Sec 4.4. However I suppose if I leave all this extra explanation in, you can probably deal with that...

Sec 4.3

\# 66. $f(x) = \frac{1}{1+x^2}$ is even so its graph is symmetric about the y-axis.

$$f'(x) = \frac{-2x}{(1+x^2)^2}$$

$$f''(x) = \frac{2(3x^2-1)}{(1+x^2)^3}$$



$-\frac{1}{\sqrt{3}}$ 0 $\frac{1}{\sqrt{3}}$

f

f' + + + + + + 0 − − − − − −

f'' + + 0 − − − − − − 0 + +

f is increasing on $(-\infty, 0)$
and decreasing on $(0, \infty)$.

f is concave up on $(-\infty, -\frac{1}{\sqrt{3}})$ and on $(\frac{1}{\sqrt{3}}, \infty)$;
it is concave down on $(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$.

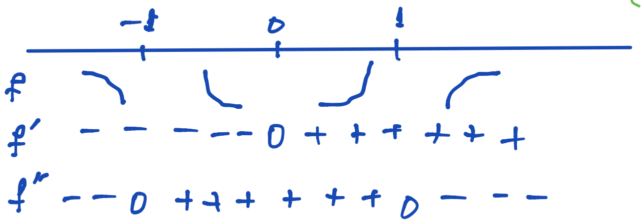
Since $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0$, the x-axis is a horizontal asymptote and there is no local or global minimum for f. However f has a unique critical point $(0,1)$ which is both a local and global minimum point. There are two inflection points, as labelled, and no vertical asymptotes.

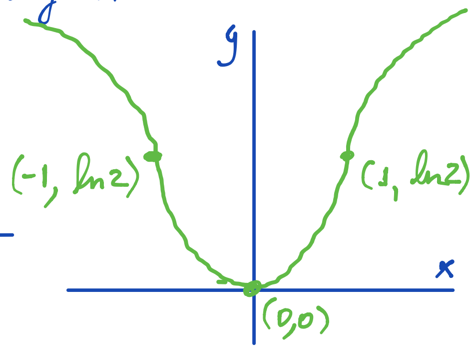
Sec 4.4

#36. $f(x) = \ln(x^2+1)$ is an even function, so its graph is symmetric about the y-axis.

$$f'(x) = \frac{2x}{x^2+1}$$

$$f''(x) = \frac{2-2x^2}{(x^2+1)^2} = \frac{2(1+x)(1-x)}{(x^2+1)^2}$$





f is decreasing on $(-\infty,0)$; increasing on $(0,\infty)$;
concave up on $(-1,1)$; and concave down on $(-\infty,-1)$ and on $(1,\infty)$.

Since $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = \infty$, f has no absolute maximum.

Its only critical point is $(0,0)$, which is both a local and absolute minimum. There is no local maximum, and there are no asymptotes. The only intercept is the origin $(0,0)$, which is both a x-intercept and a y-intercept.

Sec 4.4

#38. $f(x) = x - 3x^{2/3}$

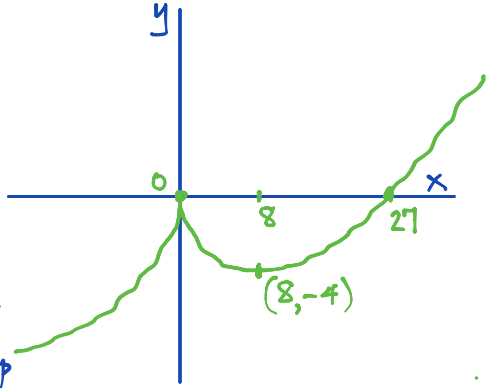
$f'(x) = 1 - 2x^{-1/3}, \quad x \neq 0$

$f''(x) = \frac{2}{3}x^{-4/3}, \quad x \neq 0.$

f has no obvious symmetry; it is neither even nor odd.

Critical points are at 0 (where f' is undefined) and 8 (where $f'(8) = 0$).

		0		8	
f					
f'	+ + +		− − −	0	+ + + +
f''	+ + +		+ + + +		+ + + +



f is increasing on $(-\infty, 0)$ and on $(8, \infty)$; decreasing on $(0, 8)$; and concave up on both $(-\infty, 0)$ and $(0, \infty)$.

It has no inflection points. It has a local minimum point $(8, -4)$ and a local maximum point $(0, 0)$. There are no asymptotes and no absolute extrema since $\lim_{x \to -\infty} f(x) = -\infty$, $\lim_{x \to \infty} f(x) = \infty$.

Note that f is continuous everywhere, but not differentiable at 0, where the graph has a cusp.

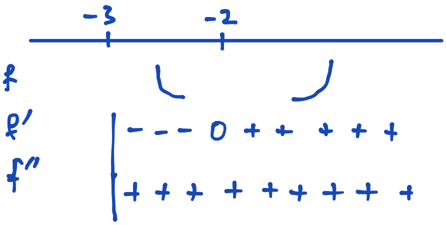
f has two x-intercepts (at 0 and 27). Its y-intercept is at 0.

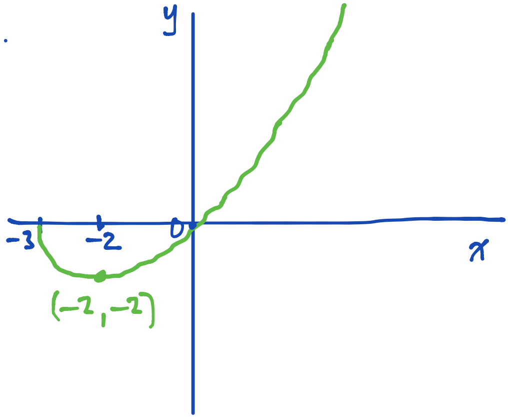
Sec 4.4

#42. $f(x) = x\sqrt{x+3}\,, \quad x \geq -3.$

$$f'(x) = \sqrt{x+3} + \frac{x}{2\sqrt{x+3}} = \frac{3(x+2)}{2\sqrt{x+3}}\,, \quad x > -3.$$

$$f''(x) = \frac{3(x+4)}{4(x+3)^{3/2}}\,, \quad x > -3.$$





f is decreasing on $(-3,-2)$ and increasing on $(-2,\infty)$.

f is concave up on $(-3,\infty)$.

Note that f is continuous on $[-3,\infty)$ and differentiable on $(-3,\infty)$.

It has no inflection points.

$\lim_{x\to -3^+} f(x) = f(-3) = 0$ and $\lim_{x\to\infty} f(x) = \infty$

So there is no absolute maximum and no asymptotes.

The local minimum point $(-2,-2)$ is also the absolute minimum.