Sec 4.2

#21. (a) Yes, the conditions of the Mean Value Theorem are satisfied: f is continuous and differentiable since it is a polynomial.

(b) $\frac{f(2)-f(-1)}{2-(-1)} = \frac{3-6}{3} = -1$; $f'(x) = -2x$.

Solve $f'(c) = -2c = -1$ to obtain $c = \frac{1}{2}$, the unique point in the interval $(-1,2)$ where $f'(c)$ equals the average rate of change of f on $[-1,2]$.

#22. (a) Yes, the conditions of the Mean Value Theorem are satisfied: f is continuous and differentiable since it is a polynomial.

(b) $\frac{f(1)-f(0)}{1-0} = \frac{-1-0}{1-0} = -1$, $f'(x) = 3x^2-4x$. To find points where $f'(c) = 3c^2-4c = -1$, solve $0 = 3c^2-4c+1 = (3c-1)(c-1)$.

The roots of this equation are $\frac{1}{3}$ and 1; however we require $0 < c < 1$. So the only solution is $c = \frac{1}{3}$.

#23. (a) No; the conditions required by the Mean Value Theorem are not satisfied. Although f is continuous, it is not differentiable at 0.

#24. (a) No; this function is neither continuous nor differentiable at 1.

#25. (a) Yes; the exponential function is both continuous and differentiable.

(b) $\frac{f(1)-f(0)}{1-0} = \frac{e-1}{1} = e-1$, $f'(x) = e^x$. We must find c such that $e^c = e-1$. The unique solution is $c = \ln(e-1) \approx 0.541$, a point in the open interval $(0,1)$.