Sec 3.8

#20. (a) $\frac{d}{dx}\tan(xy) = \frac{d}{dx}(x+y)$

$\sec^2(xy)\left(x\frac{dy}{dx}+y\right) = 1+\frac{dy}{dx}$

$\left(x\sec^2(xy)-1\right)\frac{dy}{dx} = 1-y\sec^2(xy)$

$\frac{dy}{dx} = \frac{1-y\sec^2(xy)}{x\sec^2(xy)-1}$

(b) $\left.\frac{dy}{dx}\right|_{(0,0)} = \frac{1-0}{0-1} = -1.$

#27. $\sin x + \sin y = y$

$\cos x + (\cos y)\frac{dy}{dx} = \frac{dy}{dx}$

$\cos x = (1-\cos y)\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{\cos x}{1-\cos y}$

#28. $y = xe^y$

$\frac{dy}{dx} = e^y + xe^y\frac{dy}{dx}$

$(1-xe^y)\frac{dy}{dx} = e^y$

$\frac{dy}{dx} = \frac{e^y}{1-xe^y} = \frac{e^y}{1-y}$

#45. (a) $\sin 0 + 5\cdot 0 = 0 = 0^2$ so $(0,0)$ lies on the curve.

(b) $(\cos y)\frac{dy}{dx} + 5 = 2y\frac{dy}{dx}$

$5 = (2y-\cos y)\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{5}{2y - \cos y}$$

$$\left.\frac{dy}{dx}\right|_{(0,0)} = \frac{5}{0-1} = -5$$

The tangent line at $(0,0)$ is $y-0 = -5(x-0)$,
i.e. $y = -5x$.

#48. (a) $(-1)^4 - (-1)^2 \cdot 1 + 1^4 = 1-1+1 = 1$, so $(-1,1)$ lies on the curve.

(b) $x^4 - x^2 y + y^4 = 1$

$$4x^3 - 2xy - x^2\frac{dy}{dx} + 4y^3\frac{dy}{dx} = 0$$

$$(4y^3 - x^2)\frac{dy}{dx} = 2xy - 4x^3$$

$$\frac{dy}{dx} = \frac{2xy - 4x^3}{4y^3 - x^2}$$

$$\left.\frac{dy}{dx}\right|_{(-1,1)} = \frac{2-4}{4-1} = -\frac{2}{3}$$

The tangent line at $(-1,1)$ is $y-1 = -\frac{2}{3}(x+1)$,
i.e. $y = -\frac{2}{3}x + \frac{1}{3}$.

#67. $9x^2 + y^2 - 36x + 6y + 36 = 0 \quad (*)$

$$18x + 2y\frac{dy}{dx} - 36 + 6\frac{dy}{dx} = 0$$

$$(2y+6)\frac{dy}{dx} = 36 - 18x.$$

$$\frac{dy}{dx} = \frac{36-18x}{2y+6} = \frac{9(2-x)}{y+3}$$

(a) For horizontal tangent lines, $\frac{dy}{dx} = 0$ so $x=2$. Substituting into $(*)$ gives $y^2 + 6y = y(y+6) = 0$ so the points $(0,0)$ and $(0,-6)$ have horizontal tangent lines. These are the lines $y=0$ and $y=-6$ respectively.

(b) For vertical tangent lines, $\frac{dy}{dx}$ becomes undefined. This occurs when $y = -3$. Substituting this into (*) gives

$$9x^2 - 36x + 27 = 9(x-1)(x-3) = 0$$ so vertical tangent lines are at the points $(1,-3)$ and $(3,-3)$. These are the lines $x = 1$ and $x = 3$ respectively.