

Calculus I

Sec 3.1 Solutions to Practice Problems 3

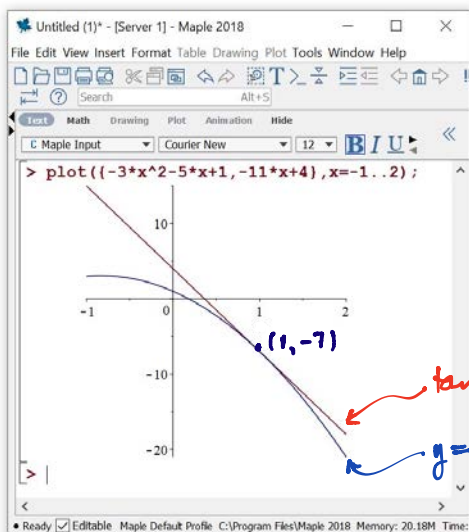
#3. The slope of any line is the rate of change of height (y) with respect to horizontal distance (x). In the case of tangent lines, we may take it as a matter of definition that the tangent line is the line through the point of tangency, having slope given by the instantaneous rate of change of height with respect to horizontal distance.

#10. The line has equation $y - 4 = 7(x + 2)$, i.e. $y = 7x + 18$.

$$\begin{aligned} \#16. (a) f'(1) &= \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1} \frac{-3x^2 - 5x + 1 + 7}{x - 1} = \lim_{x \rightarrow 1} \frac{(-3x - 8)(x - 1)}{x - 1} \\ &= \lim_{x \rightarrow 1} (-3x - 8) = -11. \end{aligned}$$

(b) $y + 7 = -11(x - 1)$, i.e. $y = -11x + 4$.

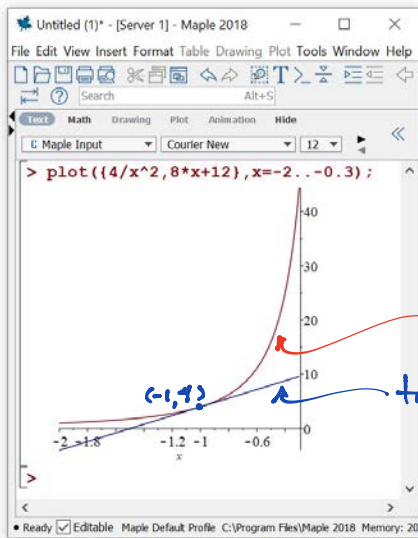
(c)



$$\# 18. (a) f'(1) = \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x-1} = \lim_{x \rightarrow 1} \frac{\frac{4}{x^2} - 4}{x-1} = \lim_{x \rightarrow 1} \frac{4(1-x^2)}{x^2(x-1)} = \lim_{x \rightarrow 1} \frac{4(1-x)}{x^2} = 8.$$

$$(b) y-4 = 8(x-1), \text{ i.e. } y = 8x + 12.$$

(c)



$$y = f(x) = \frac{4}{x^2}$$

$$\text{tangent line } y = 8x + 12$$

37.

$$\begin{aligned} (a) f'\left(\frac{1}{4}\right) &= \lim_{x \rightarrow \frac{1}{4}} \frac{f(x) - f\left(\frac{1}{4}\right)}{x - \frac{1}{4}} = \lim_{x \rightarrow \frac{1}{4}} \frac{\frac{1}{\sqrt{x}} - 2}{x - \frac{1}{4}} \cdot \frac{1\sqrt{x}}{1\sqrt{x}} = \lim_{x \rightarrow \frac{1}{4}} \frac{1(1-2\sqrt{x})}{(4x-1)\sqrt{x}} \\ &= \lim_{x \rightarrow \frac{1}{4}} \frac{4(1-2\sqrt{x})}{(2\sqrt{x}+1)(2\sqrt{x}-1)\sqrt{x}} = \lim_{x \rightarrow \frac{1}{4}} \frac{-4}{(2\sqrt{x}+1)\sqrt{x}} = \frac{-4}{2 \cdot \frac{1}{2}} = -4. \end{aligned}$$

$$(b) y-2 = -4\left(x-\frac{1}{4}\right) = -4x+1, \text{ i.e. } y = -4x+3.$$