Solutions to Practice Problems 2.

Sec 2.3

#5. Since the two functions $f(x) = \frac{x^2-7x+12}{x-3} = \frac{(x-3)(x-4)}{x-3}$ and $g(x) = x-4$ have the same value everywhere except at $x=3$ (where f is undefined and g is defined), they have the same limit as $x \to 3$. (The graph of g is a straight line, and the graph of f is almost the same line, but missing the point $(3,-1)$; on the other hand, the missing point does not affect the limit of either f or g at any point). So

$$\lim_{x\to 3} \frac{x^2-7x+12}{x-3} = \lim_{x\to 3} (x-4) = -1.$$

#8. $\lim_{x\to 1} \frac{f(x)}{h(x)} = \frac{8}{2} = 4$

#11. $\lim_{x\to 1} \frac{f(x)}{g(x)-h(x)} = \frac{8}{3-2} = 8.$

#47. $\lim_{h\to 0} \frac{\sqrt{16+h}-4}{h} = \lim_{h\to 0} \frac{\sqrt{16+h}-4}{h} \cdot \frac{\sqrt{16+h}+4}{\sqrt{16+h}+4} = \lim_{h\to 0} \frac{h}{h(\sqrt{16+h}-4)}$

$= \lim_{h\to 0} \frac{1}{\sqrt{16+h}+4} = \frac{1}{4+4} = \frac{1}{8}.$

#60. $\lim_{x\to 0} \frac{\sin 2x}{\sin x} = \lim_{x\to 0} \frac{2\sin x\cos x}{\sin x} = \lim_{x\to 0} 2\cos x = 2\cdot 1 = 2.$

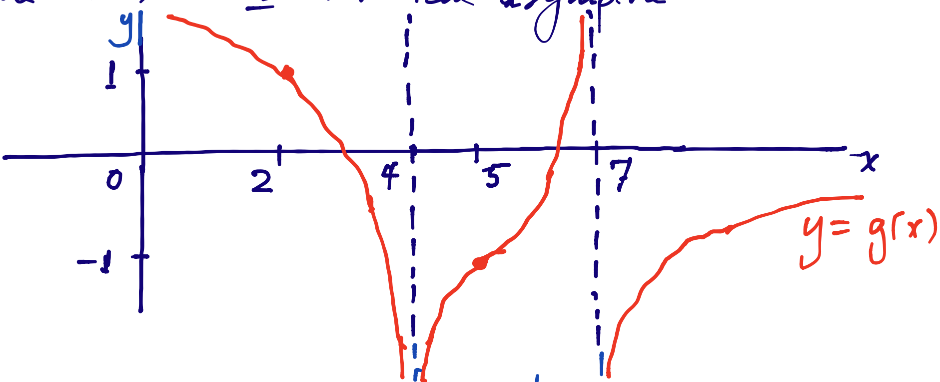
Sec 2.4

#8. (a) $\lim_{x\to 2^-} g(x) = \infty$ (b) $\lim_{x\to 2^+} g(x) = -\infty$ (c) $\lim_{x\to 2} g(x)$ does not exist.

(d) $\lim_{x\to 4^-} g(x) = -\infty$ (e) $\lim_{x\to 4^+} g(x) = -\infty$ (f) $\lim_{x\to 4} g(x) = -\infty$.

#15. The denominator of $f(x)$ vanishes at $x \in \{1,2\}$ and so f is undefined at those points. Since $\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{x-1}{x-2} = 0$, there is no vertical asymptote at $x=1$. However since $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{x-1}{x-2} = \infty$ and $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{x-1}{x-2} = -\infty$, the line $x=2$ is a vertical asymptote.

#18.



#30. (a) This limit is undefined since the denominator is undefined for $1<x<4$. (The expression inside the square root is negative.)

(b) $\lim_{x\to 1^-} \frac{x-3}{\sqrt{(x-1)(x-4)}} = -\infty$. (The denominator is positive, approaching zero from above; the numerator is negative approaching -2.)

(c) The two-sided limit is undefined since (a) is undefined.

#43. $\lim_{\theta\to 0} \frac{2+\sin\theta}{1-\cos^2\theta} = \infty$ since the numerator is positive and ≥ 1; the denomiaator is also positive and approaching zero.

#48. The denominator vanishes at $x \in \{-2, 0\}$.

Since $\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} \dfrac{\cos x}{x(x+2)} = \infty$ and $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} \dfrac{\cos x}{x(x+2)}$

$= \infty$, the y-axis $(x = 0)$ is a vertical asymptote.

Since $\lim\limits_{x \to -2^+} f(x) = \infty$ and $\lim\limits_{x \to -2^-} f(x) = -\infty$, the line $x = -2$ is also a vertical asymptote. Note that $f(x) = \dfrac{\cos x}{x(x+2)}$ where the numerator takes positive values at both $x \in \{0, -2\}$.

Also note that neither of the two-sided limits $\lim\limits_{x \to 0} f(x)$ nor $\lim\limits_{x \to -2} f(x)$ exists.