Solutions to Practice Problems 1

Sec 2.1

#8. On $[2, 2.01]$, $\frac{\Delta s}{\Delta t} = \frac{55.9584 - 56}{2.01 - 2} = -4.16$.

On $[2, 2.001]$, $\frac{\Delta s}{\Delta t} = \frac{55.995984 - 56}{2.001 - 2} = -4.016$.

On $[2, 2.001]$, $\frac{\Delta s}{\Delta t} = \frac{55.99959984 - 56}{2.001 - 2} = -4.0016$.

These values appear to be converging to a limiting value of 4. This leads us to conjecture that at time $t = 2$, the instantaneous velocity is 2.

#12. The slope of the secant line from $(2, f(2))$ to $(x, f(x))$ is

$$\frac{\Delta f(x)}{\Delta x} = \frac{f(x) - f(2)}{x - 2}.$$

(a) We tabulate the following values:

x	1.5	1.9	1.99	1.999	1.9999
$\frac{f(x) - f(2)}{x - 2}$	8.0	1.6	0.16	0.016	0.0016

(b) We conjecture that these values have a limit of 0 as x approaches 2.

(c) The answer in (b) confirms our prediction based on the graph: the green secant line approaches the red tangent line as $x \to 2$; and this tangent line is horizontal so its slope is zero.

#14. (a) $\frac{s(3)-s(0)}{3-0} = 15.3$

(b) $\frac{s(2)-s(0)}{2-0} = 20.2$

(c) $\frac{s(1)-s(0)}{1-0} = 25.1$

(d) $\frac{s(h)-s(0)}{h-0} = -4.9h + 30$

Remark: From (d) we see that the instantaneous velocity at time 0 is $\lim_{h \to 0} (-4.9h + 30) = 30$.

#20. Please make sure your calculator is set in radians mode (reviewed in Sec 1.4). Values in the second column are

1.9099, 0.14988, 0.0149999, 0.001500, 0.0001500.

We conjecture, based on these values, that the instantaneous velocity at $t=0$ is 0.