Name ....

Section ☐ **01** Earl-8am ☐ **02** Yeung-9am ☐ **03** Furtado-10am ☐ **04** Li-11am
☐ **05** Furtado-11am ☐ **06** Zhong-12noon ☐ **07** Wiseman-1:10pm ☐ **08** Yeung-2:10pm

Common Exam I

5:15–7:00pm Thursday February 21, 2019

Instructions. Indicate your name and section/instructor above. You may use a scientific non-graphing calculator—no other aids are allowed. Cell phones and other devices must be turned off and left in your backpack/bag during the exam. **Write clearly**, using good mathematical notation and showing all required steps in the space provided. Total value: 100 points.

1. *(10 points)* A graph of the function $f(x) = x\sin\left(\frac{1}{x}\right)$ is shown below.

   Indicate whether each of the following statements is true or false, by circling T or F respectively.

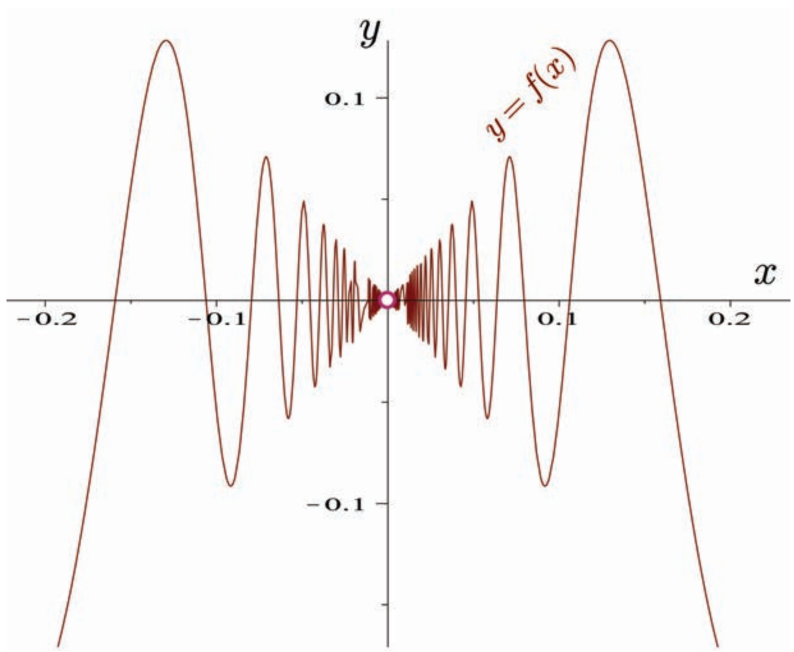
$\boxed{\text{T}}$ $\boxed{\text{F}}$ $f(0) = 0.$

$\boxed{\text{T}}$ $\boxed{\text{F}}$ $\lim_{x\to 0} f(x) = 0.$

$\boxed{\text{T}}$ $\boxed{\text{F}}$ $\lim_{x\to 0} f(x) = \infty.$

$\boxed{\text{T}}$ $\boxed{\text{F}}$ $\lim_{x\to \infty} f(x) = 0.$

$\boxed{\text{T}}$ $\boxed{\text{F}}$ $\lim_{x\to -\infty} f(x) = \infty.$



2. *(9 points)* Let $f(x) = \dfrac{x+5}{x-2}$. Complete the nine blanks below using simple numerical values, showing how one computes $f'(3)$ from the definition.

$$
\begin{aligned}
f'(3) &= \lim_{h \to 0} \frac{f(\square + h) - f(\square)}{h} \\
&= \lim_{h \to 0} \frac{\dfrac{\square + h}{\square + h} - 8}{h} \\
&= \lim_{h \to 0} \frac{(\square + h) - 8(\square + h)}{h(1+h)} \\
&= \lim_{h \to 0} \frac{\square h}{h(1+h)} \\
&= \lim_{h \to 0} \frac{\square}{1+h} \\
&= \square .
\end{aligned}
$$

3. *(12 points)* Multiple choice: Circle the correct response (A, B, C or D).

(a) The average rate of change of a function f on the interval $[0, 2]$ is

A. $f(2)-f(0)$

B. $\dfrac{f(2)-f(0)}{2}$

C. $\lim_{h\to 0} \dfrac{f(2+h)-f(h)}{h}$

D. $\lim_{h\to 0} \dfrac{f(2+h)-f(2)}{h}$

(b) The instantaneous rate of change of $f(x)$ at $x = 2$ is

A. $\lim_{h\to 0} \dfrac{f(2+h)-f(h)}{h}$

B. $\lim_{h\to 0} \dfrac{f(2+h)-f(2)}{h}$

C. $\lim_{h\to 0} \dfrac{f(2)-f(h)}{2-h}$

D. $\lim_{h\to 0} \dfrac{f(2+h)-f(h)}{2}$

(c) If an object has position $s(t)$ at time t, then its average velocity during the time interval $[0, t]$ is

A. $s'(0)$

B. $\dfrac{s(t)-s(0)}{t-0}$

C. $\lim_{t\to 0} s(t)$

D. $\lim_{h\to 0} \dfrac{s(t+h)-s(h)}{h}$

(d) If an object has position $s(t)$ at time t, then its instantaneous velocity at time $t = 0$ is

A. $s'(0)$

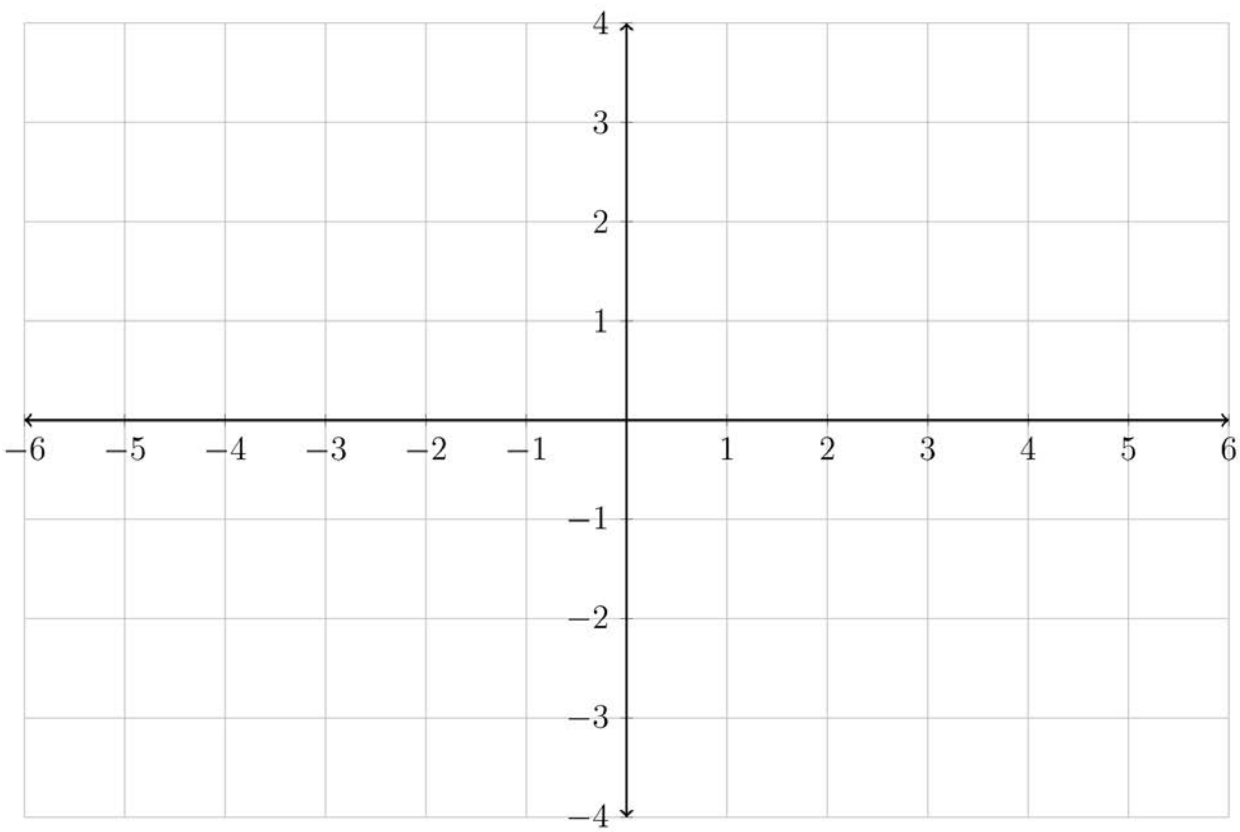
B. $\dfrac{s(t)-s(0)}{t-0}$

C. $\lim_{t\to 0} s(t)$

D. $\lim_{h\to 0} \dfrac{s(t+h)-s(h)}{h}$

4. *(12 points)* Using the axes below, sketch the graph of a function $g(x)$ satisfying the following conditions:

- $\lim_{x \to 2^+} g(x) = g(2) = 3$;
- $\lim_{x \to 2^-} g(x) = -\infty$;
- $\lim_{x \to -2^+} g(x) = \infty$;
- $\lim_{x \to -2^-} g(x) = g(-2) = 1$;
- $g(0)$ and $\lim_{x \to 0} g(x)$ are both defined, but $g(x)$ is not continuous at $x = 0$.



5. *(8 points)* The number of people in a theater at time t (in minutes) is given by a function $f(t)$. Values, recorded at 2-minute intervals, are tabulated as shown:

t	0	2	4	6	8
$f(t)$	5	7	25	61	117

(a) During the entire 8-minute interval, what was the average rate of increase of people in the theater (in people per minute)?

(b) Give a reasonable estimate for the instantaneous rate at which people were entering the theater at time $t = 5$ minutes.

6. *(8 points)*

(a) The charge for parking a passenger car at the airport during a time interval of t minutes, is $C(t)$ dollars. One driver who parks for 15 minutes is charged 5 dollars, so $C(15) = 5$. Another driver parks for 2 hours and is charged 20 dollars, so $C(120) = 20$.

Based on the information given, can we reasonably conclude that there is a time interval for which a driver will be charged exactly 12 dollars for parking? *Explain*.

(b) The temperature in a garage at time t (in hours after midnight) on a given day is $T(t)$. We are given that the temperature is 25 degrees at 5am and 35 degrees at 11am, so $T(5) = 25$ and $T(11) = 35$.

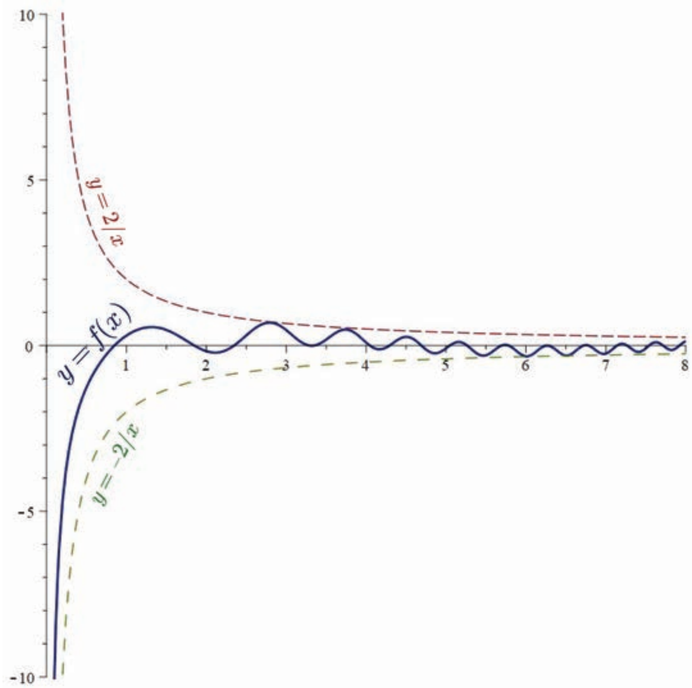
Based on the information given, can we reasonably conclude that the temperature in the garage was 30 degrees at some time that morning? *Explain*.

(c) *Name the theorem*, studied in class this semester, which describes situations under which we may draw conclusions such as those described above.

7. *(8 points)* A function $f(x)$ is known to satisfy

$$-\frac{2}{x} \leqslant f(x) \leqslant \frac{2}{x}$$

for all $x > 0$. *One possible function* f satisfying these inequalities is shown:



(a) *Using only the inequalities above*, can we determine $\lim_{x\to\infty} f(x)$ using the Squeeze Theorem? Explain, and if the limit is known, indicate its value.

(b) *Using only the inequalities above*, can we determine $\lim_{x\to 0^+} f(x)$ using the Squeeze Theorem? Explain, and if the limit is known, indicate its value.

8. *(9 points)* A function f is given by

$$f(x) = \begin{cases} 2^x, & \text{for } x < 1; \\ a, & \text{for } x = 1; \text{ and} \\ \sqrt{b+x}, & \text{for } x > 1 \end{cases}$$

where a and b are constants.

Determine values of the constants a and b such that f is continuous everywhere. Explain your work.

9. *(12 points)* Find the following limits algebraically, using the limit laws (*not using calculator estimates!*). Use proper mathematical notation, symbols, syntax, and terminology at all times.

(a) $\displaystyle\lim_{x\to 0} \frac{x^3-8}{x^2-4}$

(b) $\displaystyle\lim_{x\to 2} \frac{x^3-8}{x^2-4}$

(c) $\displaystyle\lim_{x\to \infty} \frac{x^3-8}{x^2-4}$

(d) $\displaystyle\lim_{x\to -\infty} \frac{x^3-8}{x^2-4}$

10. *(12 points)* Experiment suggests that a falling body will fall a distance of $s(t) = 16t^2$ feet in t seconds.

(a) How far will it fall between $t = 2$ and $t = 3$?

(b) What is the average velocity on the interval $2 \leqslant t \leqslant 3$?

(c) What is the average velocity on the interval $2 \leqslant t \leqslant 2 + h$? (Here h is a small positive number.)

(d) Find its instantaneous velocity at $t = 2$.

(SCRATCH WORK)

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For instructors' use only:

Question	1	2	3	4	5	6	7	8	9	10	Total
Points	10	9	12	12	8	8	8	9	12	12	100
Score											